In section 8.2 you will learn to:

- Decide whether two matrices are equal.
- Add and subtract matrices and multiply a matrix by a scalar.
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In 8.2b you will learn to:

- Multiply two matrices.
- Set up an $n \times n$ Identity matrix.

Matrix addition: A + B

If A = $[a_{ij}]$ and B = $[b_{ij}]$, then A+B = $[a_{ij} + b_{ij}]$

A and B must be the same size as is the sum.
Each element in the sum is the sum of the corresponding elements.

A + B =

$$A = \begin{bmatrix} 2 & 1 \\ 3 & -2 \\ 0 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -2 \\ 3 & 2 \\ -1 & 4 \end{bmatrix}$$

Scalar Multiplication - A scalar is a real number or constant.

cA = $[ca_{ij}]$ ie. Every element of A gets multiplied by c.

cA =

Properties of matrix multiplication:

- A + B = B + A
- A + (B + C) = (A + B) + C
- cd(A) = c(dA)
- 1·A = A
- c(A+B) = cA + cB
- (c+d)A = cA + dA

Properties of matrix addition and scalar multiplication -

- $A + B = B + A$
- $A + (B + C) = (A + B) + C$
- $cd(A) = c(dA)$
- $1 \cdot A = A$
- $c(A+B) = cA + cB$
- $(c+d)A = cA + dA$

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & -2 & 1 \\ 0 & 5 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -2 & 1 \\ 3 & 2 & 5 \\ -1 & 4 & 0 \end{bmatrix}$$

Example 1

a) $A + B =$

b) $A - B =$

c) $3A - 2B$

Matrix Multiplication

$A = [a_{ij}]$ an m x n matrix $B = [b_{ij}]$ an n x p matrix

$AB = [c_{ij}]$ an m x p matrix

where $c_{ij} =$

Example 2: Find AB if possible, then find BA

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}$$

Notice: $AB \neq BA$

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \qquad \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} =$$

The Identity matrix:

$$I = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

* Must be square
* The diagonal has all 1s $I_{jj}=1$
* Zeros in all other positions

Notice that $I \cdot A = A \cdot I = A$

$$\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} =$$

Matrix Multiplication Properties:

1. A(BC) = (AB)C
2. A(B+C) = AB + AC
3. (A+B)C = AC + BC
4. c(AB) = (cA)B = A(cB)
5. $I \cdot A = A \cdot I = A$

Example 3: Find AB if possible

$$A = \begin{bmatrix} 3 \\ -1 \\ 5 \\ 4 \end{bmatrix} \qquad B = [5 \ \ -3] + [0 \ \ 2] + [4 \ \ -1]$$

Example 4: Find AB, BA, and A^2, if possible

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$$

Put this system into matrix algebra form: $A \cdot X = C$

A is the Matrix of coefficients.

X is the matrix of variables.

C is the matrix of constants.

$$\begin{aligned} x - y + 4z &= 17 \\ x + 3y &= -11 \\ 2y + 5z &= 0 \end{aligned}$$