Linear and non-linear systems of equations

A system of equations is simply a set of two or more equations in two or more variables that we solve simultaneously.

A system of linear equations in two variables has three possible outcomes:
You already know two strategies to solve two equations in two unknowns.

1. Graphically - Not reliable, but useful.

2. Substitution - A method that will always work.

1) Solve this set of linear equations using both of the methods.

\[ \begin{align*}
\text{(1)} & \quad x - y = -4 \\
\text{(2)} & \quad x + 2y = 5
\end{align*} \]

\[ \begin{align*}
\text{Graphically:} & \quad y = x + 4 \\
& \quad y = \frac{1}{2}x + \frac{5}{2}
\end{align*} \]

\[ \begin{align*}
\text{Substitution:} & \quad x = y - 4 \\
& \quad y - 4 + 2y = 5 \\
& \quad 3y - 4 = 5 \\
& \quad 3y = 9 \\
& \quad y = 3 \\
& \quad x = 3 - 4 = -1
\end{align*} \]

\[ (3, 1, 3.2) \quad \frac{3}{1} \quad (1, 3) \]
2) Solve using substitution.

3x + y = 2
x^3 - 2 + y = 0

\[ \begin{align*}
1) & \quad y = -3x + 2 \\
2) & \quad x^3 - 2 - 3x + 2 = 0 \\
& \quad x^3 - 3x = 0 \\
& \quad x(x^2 - 3) = 0 \\
& \quad x = 0, x = \pm \sqrt{3} \\
& \quad x = 0, \quad x = \pm \sqrt{3}
\end{align*} \]

\[ \begin{align*}
1) & \quad y = -3(0) + 2 = 2, \quad (0, 2) \\
& \quad x = \sqrt{3}, \quad y = -3\sqrt{3} + 2, \quad (\sqrt{3}, -3\sqrt{3} + 2) \\
& \quad x = -\sqrt{3}, \quad y = 3(\sqrt{3}) + 2 = 3\sqrt{3} + 2, \quad (-\sqrt{3}, 3\sqrt{3} + 2)
\end{align*} \]
3) Solve by graphing.

\[ 2x - y + 3 = 0 \]
\[ x^2 + y^2 - 4x = 0 \]

Graph:

- Line: \( y = 2x + 3 \)
- Circle: \((x-h)^2 + (y-k)^2 = r^2\) with center \((h,k)\) and radius \(r\) in radians

Solve:

\[
\begin{align*}
\text{Line:} & \quad y = 2x + 3 \\
\text{Circle:} & \quad x^2 + y^2 - 4x = 0 \\
& \quad (x^2 - 4x) + y^2 = 0 \\
& \quad (x^2 - 4x + 4) + y^2 = 4 \\
& \quad (x-2)^2 + y^2 = 4 \\
& \quad \Rightarrow \text{N.S.}
\end{align*}
\]
4) Solve

1) \( y = (x+1)^3 \)

2) \( y = \sqrt{x-1} \)

1) Base graph \( y = x^3 \)
   Shift graph left 1

2) Base graph \( y = \sqrt{x} \)
   Shift graph 1 unit to right

N.S.
5) Solve

\[ y = x^3 - 2x^2 + x - 1 \]
\[ y = -x^2 + 3x - 1 \]

- \[ x = 0 \]
  \[ y = 0 + 0 - 1 = -1 \]
  \[ \boxed{(0, -1)} \]

- \[ x = -1 \]
  \[ y = (-1)^2 + 3(-1) - 1 = 1 - 3 - 1 = -5 \]
  \[ \boxed{(-1, -5)} \]

- \[ x = 2 \]
  \[ y = -2 + 3(2) - 1 = -2 + 6 - 1 = 3 \]
  \[ \boxed{(2, 3)} \]

\[ -x^2 + 3x - 1 = x^3 - 2x^2 + x - 1 \]
\[ \boxed{0 = x^2 - x - 2} \]
\[ \boxed{0 = x(x + 1)(x - 2)} \]

\[ x = 0 \]
\[ x + 1 = 0 \]
\[ x - 2 = 0 \]
\[ x = -1 \]
\[ x = 2 \]