In section 3.3 you will learn to:

- Use properties to evaluate or rewrite logarithmic expressions.
- Use properties of logarithms to expand or condense logarithmic expressions.
- Use the change of base formula to rewrite and evaluate logarithmic expressions.
- Use logarithmic functions to model and solve real-life problems.

Properties of Logarithms

Your calculator has only two keys that compute logarithmic values.

log x means $\log_{10}x$

ln x means $\log_e x$

Suppose you need to compute a logarithm in some other base, a

$\log_a x = y$

Change of base formula:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Examples:

a) $\log_2 254 =$

b) $\log_6 0.008 =$

Since a logarithm is an exponent, the properties of logarithms are just like the properties of exponents.

	Exponents	Logarithms
	$a^0=1$	
	$a^1= a$	
Product:	$a^m \cdot a^n =$	$\log_a(uv) =$
Quotient:	$\frac{a^m}{a^n} =$	$\log_a \frac{u}{v} =$
Power:	$(a^m)^n =$	$\log_a u^n =$

Inverse properties:

One-to-one properties:

Let's apply the properties of logarithms.

a) $\log_4 5 + \log_4 6 =$

b) $\log (12a) - \log (2a) =$

c) $\log_4 x^4 =$

d) $e^{\ln(5x)} =$

e) $\log 10^{(x+2)} =$

In solving equations, it will be helpful to expand and condense logarithmic expressions.

Expand these:

a) $\log_4 5x^3y =$

b) $\ln \frac{\sqrt{3x-5}}{7} =$

c) $\log \left(\frac{b^3}{1+a^2}\right)^5 =$

Condense these into a single logarithmic expression:

a) $1/2 \log x + 3 \log (x+1) =$

b) $2 \ln (x+2) - \ln x =$

Suppose we know that $\log_b 2 = 0.41$ and $\log_b 3 = 0.54$, use the properties of logarithms to find:

a) $\log_b 6 =$

b) $\log_b 2/9 =$

c) $\log_b 8\sqrt{3} =$

Logarithms are useful in reporting a broad range of data by converting it into a more manageable form. Consider the intensity of earthquakes.

Let I_o = the intensity of a "standard" earthquake that is agreed upon as minimal (barely detectable.)

Let I = The intensity of a much larger earthquake.

The magnitude M of the latter quake I relative to I_o is defined by

$$M = \log \frac{I}{I_o}$$

You may have heard of the *Richter scale* that measures the intensity of an earthquake.

What is the magnitude M of an earthquake measured to be 10,000 times more intense than a standard quake?

Example:

On October 17, 1989 a major earthquake struck the San Francisco Bay area only minutes before Game 3 of the World Series in Candlestick Park. Its intensity was measured as 7.l on the Richter scale.

How many times more intense was it than a minimal quake?

a) 12, 500 times more intense?

b) 1,250,000 time more intense?

c) 12,500,000 times more intense?