Properties of Logarithms

In section 3.3 you will learn to:

- Use properties to evaluate or rewrite logarithmic expressions.
- Use properties of logarithms to expand or condense logarithmic expressions.
- Use the change of base formula to rewrite and evaluate logarithmic expressions.
- Use logarithmic functions to model and solve real-life problems.

3.3 Properties of Logarithms

Properties of Logarithms

Your calculator has only two keys that compute logarithmic values.

$\log x$ means $\log_{10} x$

$\ln x$ means $\log_e x$

Suppose you need to compute a logarithm in some other base, a

$\log_a x = y$

$a^y = x$

$\log(a^y) = \log x$

$y \log a = \log x$

$y = \frac{\log x}{\log a} = \log_a x$

Change of base formula:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Examples:

a) $\log_2 254 = \frac{\log 254}{\log 2} \approx 7.989$

b) $\log_6 0.008 = \frac{\ln 0.008}{\ln 6} \approx -2.695$

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3.3 Properties of Logarithms

Since a logarithm is an exponent, the properties of logarithms are just like the properties of exponents.

	Exponents	Logarithms
	$a^0=1$	$\log_a 1 = 0$
	$a^1 = a$	$\log_a a = 1$
Product:	$a^m \cdot a^n = a^{m+n}$	$\log_a(uv) = \log_a u + \log_a v$
Quotient:	$\frac{a^m}{a^n} = a^{m-n}$	$\log_a \frac{u}{v} = \log_a u - \log_a v$
Power:	$(a^m)^n = a^{mn}$	$\log_a u^n = n(\log_a u)$

Inverse properties: $\log_a(a^x) = x$

One-to-one properties: $\log_a x = \log_a y \Longrightarrow x = y$

Let's apply the properties of logarithms.

a) $\log_4 5 + \log_4 6 =$ $\log_4 30$

b) $\log(12a) - \log(2a) =$ $\log\left(\frac{12a}{2a}\right) = \log 6$

c) $\log_4 x^4 =$ $4\log_4 x$

d) $e^{\ln(5x)} =$ $5x$

e) $\log 10^{(x+2)} =$ $(x+2)\cancel{\log 10} = \underline{x+2}$

$\log_{10} 10^{(x+2)} = \underline{x+2}$

3.3 Properties of Logarithms

In solving equations, it will be helpful to expand and condense logarithmic expressions.

Expand these:

a) $\log_4 5x^3y$ = $\log_4 5 + \log_4 (x^3) + \log_4 y =$

$= \log_4 5 + 3\log_4 x + \log_4 y$

b) $\ln \dfrac{\sqrt{3x-5}}{7}$ = $\ln \sqrt{3x-5} - \ln 7 =$

$= \frac{1}{2}\ln(3x-5) - \ln 7$.

c) $\log\left(\dfrac{b^3}{1+a^2}\right)^5$ = $5\log\left(\dfrac{b^3}{1+a^2}\right) =$

$= 5\left[\log b^3 - \log(1+a^2)\right] =$

$= 5\left[3\log b - \log(1+a^2)\right]$.

Condense these into a single logarithmic expression:

a) $\frac{1}{2} \log x + 3 \log (x+1) = \log x^{1/2} + \log (x+1)^3 =$

$= \log\left(\sqrt{x}\,(x+1)^3\right)$

b) $2 \ln (x+2) - \ln x = \ln (x+2)^2 - \ln x =$

$= \ln \frac{(x+2)^2}{x}$

Suppose we know that $\log_b 2 = 0.41$ and $\log_b 3 = 0.54$, use the properties of logarithms to find:

a) $\log_b 6 =$ $\log_b(2 \cdot 3) = \log_b 2 + \log_b 3 =$

$= 0.41 + 0.54 = 0.95$

$(9 = 3^2)$

b) $\log_b 2/9 =$ $\log_b 2 - \log_b 9 = \log_b 2 - 2\log_b 3 =$

$= 0.41 - 2(0.54) = 0.41 - 1.08 = -0.67$

c) $\log_b 8\sqrt{3} =$ $\log_b 8 + \log_b \sqrt{3} =$

$= \log_b(2^3) + \log_b 3^{1/2} =$

$= 3\log_b 2 + \frac{1}{2}\log_b 3 = 3(0.41) + \frac{1}{2}(0.54) =$

$= 1.23 + 0.27 = 1.50$

3.3 Properties of Logarithms

Logarithms are useful in reporting a broad range of data by converting it into a more manageable form. Consider the intensity of earthquakes.

Let I_0 = the intensity of a "standard" earthquake that is agreed upon as minimal (barely detectable.)

Let I = The intensity of a much larger earthquake.

The magnitude M of the latter quake I relative to I_o is defined by

$$M = \log \frac{I}{I_o}$$

You may have heard of the *Richter scale* that measures the intensity of an earthquake.

What is the magnitude M of an earthquake measured to be 10,000 times more intense than a standard quake?

I_0 $\quad$ $I = 10{,}000 I_0$

$$M = \log \frac{10{,}000 \cancel{I_0}}{\cancel{I_0}} = \log 10{,}000 = 4.0$$

Example:

On October 17, 1989 a major earthquake struck the San Francisco Bay area only minutes before Game 3 of the World Series in Candlestick Park. Its intensity was measured as 7.I on the Richter scale.

How many times more intense was it than a minimal quake?

a) 12, 500 times more intense?

b) 1,250,000 time more intense?

c) 12,500,000 times more intense?

$\boxed{M = 7.1}$

$M = \log \frac{I}{I_0}$

$7.1 = \log_{10} \frac{I}{I_0}$

$10^{7.1} = I/I_0$

$I = \underline{10^{7.1}}(I_0) = \underline{12{,}500{,}000 I_0}$