Logarithmic Functions and Their Graphs

In section 3.2 you will learn to:
  • Recognize, evaluate and graph logarithmic functions with whole number bases.
  • Recognize, evaluate and graph natural logarithmic functions.
  • Evaluate logarithms without using a calculator.
  • Use logarithmic functions to model and solve real-life problems.

Review: Exponential Functions

\[ f(x) = 2^x \]

What is the inverse of this function?

Asymptotes:
Domain:
Range:
Evaluating logarithmic functions:

**Exponential form** is equivalent to **logarithmic form**

\[ b^x = y \quad \log_b y = x \]

\[ 8^{-1} = \frac{1}{8} \quad \log_8 \left(\frac{1}{8}\right) = -1 \]

**Example 1:** Notice that a logarithm is always equal to an exponent.

Determine the answer and write each one in the other form.

1. \(10^x = \)
2. \(\log_{10} \left(\frac{1}{27}\right) =\)
3. \((\frac{9}{100})^{1/2} = \)
4. \(\log_{\sqrt{2}} (2\sqrt{2}) =\)
5. \(\log_{10}(1) =\)

**Example 2:** Find the value of \(F(x)\) in each of the following if \(F(x) = \log_a x\)

a) \(F(8) =\)

b) \(F(64) =\)

c) \(F(8^{1/9}) =\)

d) \(F(2) =\)

e) \(F(0) =\)
Evaluating logarithmic expressions on a calculator:

Base 10 logarithms are called common logarithms. They are written (without base) as \( \log x = \log_{10} x \).

\[
\log 1000 =
\]

\[
\log .001 =
\]

\[
\log (1) =
\]

\( \log 15 \) is asking the question, 10 to what power will yield 15? Your calculator will tell you this is about 1.176.

Base e logarithms are called natural logarithms. They are written as \( \log_e x = \ln x \) (the natural log of x). You may want to write them as an exponential expression to evaluate these.

\[
\ln (e^3) =
\]

\[
\ln (1/e) =
\]

\[
\ln (e^2) =
\]

\( \ln(100) \) is asking what power of e will yield 100. Your calculator will tell you this is about 4.605

Example 3:
Use a calculator to evaluate these logs to four significant digits:

\[
\log 72 =
\]

\[
\log 100.000387 =
\]

\[
\ln 218 =
\]

\[
\log_{10} 10 =
\]

Do these without a calculator, then check with a calculator.

\[
\log 100 =
\]

\[
\ln e^5 =
\]

\[
\log 0 =
\]

\[
\ln 1 =
\]
Four initial properties of logarithms:

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a a^x = x$  Inverse property
4. If $\log_a x = \log_a y$ then $x = y$  One-to-one property

Example 4: Evaluate these:

$$\log_5 1 =$$

$$\log_6 6 =$$

$$\log_2 2^{1.7} =$$

$$\ln e^{1.2} =$$

Finally, suppose $\log_a x = \log_a 100$. What can you conclude?

Graphs of logarithmic functions:

$f(x) = \log_2 x \quad g(x) = \log_{(1/2)} x$

Asymptotes:
Domain:
Range:
Transformations of logarithmic functions:

\[ f(x) = 2 + \ln x \quad g(t) = \ln (t-4) \]

Example 5: An application of logarithms:

Scientific studies show that in many cases, human memory of certain information seems to deteriorate over time and can be modeled by decreasing logarithmic functions. For example, suppose a student learns to speak French so well that on an initial exam she scores 90. Over time and without practice her score on comparable exams decreases.

\[ s(t) = 90 - 12\log_2(t+1) \]

a) What is her score on the initial exam?

b) What is her score after 2 days? After 8 days?