2.6 RATIONAL FUNCTIONS

In this section you will learn how to:

• Find the domain of rational functions
• Find horizontal, vertical and slant asymptotes of rational functions
• Analyze and sketch the graph of a rational function
• Use rational functions to model and solve real-life problems
A rational function is \[ Q(x) = \frac{N(x)}{D(x)} \]

where \( N(x) \) is a polynomial function of any degree and \( D(x) \) must be a polynomial of degree 1 or greater.

The Numerator determines the roots and the Denominator determines the vertical asymptotes.
Vertical Asymptotes are caused by zero values in the denominator.

A look at $y = \frac{1}{x}$ and some transformations
2.6 Rational Functions

January 19, 2011

The numerator tells us the roots (x-intercepts) of the function. To find the y-intercept, let x=0.

\[ y = \frac{2x - 3}{(x-1)(x+2)} \]

\[ y = \frac{2x(x+2)}{(x-1)(x+3)} \]

roots (x-intercepts) \( N(x) = 0 \)

\[ y = \frac{2x - 3}{(x-1)(x+2)} \]

When \( x = 0 \)

\[ y = \frac{-3}{(-1)(2)} = \frac{3}{2} \]

\[ y = \frac{0(2)}{(-1)(2)} = 0 = 0 \]
End behavior is determined by the quotient of the leading terms.

\[ y = \frac{2x^2}{x^2} = \frac{2}{x} \quad y = \frac{2x^2}{x^2} = 2 \]

\[ y = \frac{2x - 3}{(x-1)(x+2)} \quad y = \frac{2x(x+2)}{(x-1)(x+3)} \]

\[ \text{EB: H.A \ y = 0} \quad \text{EB \ y = 2} \]

How do we know what the function looks like? We need to make a sign line:
What happens if there is a common factor in the numerator and the denominator?

\[ y = \frac{x^3 - 3x^2 + 2x}{x^2 - 1} \]

\[ y = \frac{x(x^2-3x+2)}{(x-1)(x+1)} \]

\[ y = \frac{x(x-2)(x-1)}{(x-1)(x+1)} \]

hole in the function:

\[ x = 1 \]

\[ \frac{1(-1)}{2} = -\frac{1}{2} \]

V. Asymptotes

Denom = 0

\[ x+1 = 0 \quad x = -1 \]

End Behavior

\[ y = \frac{x^2 - 2x}{x+1} \]

\[ x \rightarrow -3 \]

asymptote \[ y = x - 3 \]

Roots:

\[ x = 0 \]

\[ x - 2 = 0 \]

\[ (0,0) \quad (2,0) \]

y-int

\[ x = 0 \]

\[ \frac{0(-2)}{1} = 0 \]
2.6 Rational Functions

January 19, 2011

IN SUMMARY:

Factor numerator and denominator

Reduce any common factors and note the hole(s) in the function.

Determine $x$ and $y$ intercepts.

Determine vertical asymptotes.

Determine end behavior.

Make a sign line: