Section 2.4 Complex Numbers

- Use the imaginary unit, *i* to write complex numbers
- Add, subtract, multiply and divide complex numbers in standard form (a+b*i*)
- Use complex conjugates to write the quotient of two complex numbers in standard form
- Find complex solutions to polynomial equations

The solution to this equation is simple: $x^2 - 9 = 0$

The solution to this equation is not so simple:
$x^2 + 9 = 0$

A new term defined: $\sqrt{-1}$ is defined to be i .

So $\sqrt{-4}$ =

$\sqrt{-8}$ =

$\sqrt{-7}$ =

Let's look at some powers of i :

i^2 =

i^3 =

i^4 =

i^5 =

i^6 =

We see a pattern developing, so can find any power of i .

i^{38} =

i^{65} =

i^{321} =

A complex number a + bi has a real part a

and an imaginary part, bi.

Operations on these are not surprising.

To add two complex numbers, add the real parts and add the imaginary parts.
Subtraction is the same idea:

$(3-2i) + (5-4i) =$

$(3-2i) - (5-4i) =$

To multiply two complex numbers, you do what you would when multiply two binomials:

$(3-2i) \cdot (5-4i) =$

Division of two complex numbers is a bit trickier.
We need to introduce another term.

The <u>conjugate</u> of $a+ bi$ is $a-bi$.

The <u>conjugate</u> of $a-bi$ is $a+bi$.

Write a conjugate for each of these:

$-3 + 4i$ $2 - 5i$ $2 + i$

Now multiply each of the conjugate pairs above, something amazing happens.

When you multiply a complex number by its conjugate, the result is a real number.

Now we can learn how to divide two complex numbers. To divide $a+bi$ by $c+di$, multiply the numerator and denominator by the conjugate of the denominator.

$$(3-2i) \div (5-4i) = \frac{(3-2i)}{(5-4i)}$$

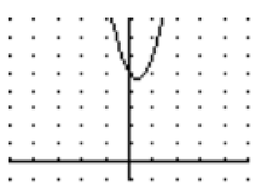
Now we can write complex solutions to quadratic equations. Solve for the roots of each of these simplifying the radical expression as much as possible.

a) $3x^2 - 2x + 5 = 0$

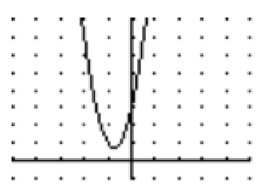
b) $4x^2 + 6x + 3 = 0$

What does this mean about the x-intercepts of the graphs of these functions?

a) $y = 3x^2 - 2x + 5$



b) $y = 4x^2 + 6x + 3$



If the solutions to a quadratic equation are complex, then the quadratic does not have x-intercepts. It will not cross the x-axis.

Roots of a quadratic:

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Are the roots of these real?

If they are real, are they rational or irrational?

a) $y = 3x^2 - x + 5$

b) $y = 3x^2 + 8x - 5$

c) $y = x^2 - x + 2$

d) $y = 3x^2 - 8x - 3$