Section 2.4 Complex Numbers

- Use the imaginary unit, $i$, to write complex numbers
- Add, subtract, multiply and divide complex numbers in standard form ($a+bi$)
- Use complex conjugates to write the quotient of two complex numbers in standard form
- Find complex solutions to polynomial equations

The solution to this equation is simple: $x^2 - 9 = 0$

The solution to this equation is not so simple: $x^2 + 9 = 0$

A new term defined: $\sqrt{-1}$ is defined to be $i$.

So $\sqrt{-4} =$

$\sqrt{-8} =$

$\sqrt{-7} =$
Let's look at some powers of $i$:

\[ i^2 = \]
\[ i^3 = \]
\[ i^4 = \]
\[ i^5 = \]
\[ i^6 = \]

We see a pattern developing, so can find any power of $i$.

\[ i^{38} = \]
\[ i^{65} = \]
\[ i^{321} = \]

A complex number $a + bi$ has a real part $a$ and an imaginary part, $bi$.

Operations on these are not surprising.

To add two complex numbers, add the real parts and add the imaginary parts.
Subtraction is the same idea:

\[ (3-2i) + (5-4i) = \]
\[ (3-2i) - (5-4i) = \]

To multiply two complex numbers, you do what you would when multiply two binomials:

\[ (3-2i) \cdot (5-4i) = \]
Division of two complex numbers is a bit trickier. We need to introduce another term.

The conjugate of $a + bi$ is $a - bi$.

The conjugate of $a - bi$ is $a + bi$.

Write a conjugate for each of these:

$-3 + 4i$  
$2 - 5i$  
$2 + i$

Now multiply each of the conjugate pairs above, something amazing happens.

When you multiply a complex number by its conjugate, the result is a real number.

Now we can learn how to divide two complex numbers. To divide $a + bi$ by $c + di$, multiply the numerator and denominator by the conjugate of the denominator.

$$\frac{3 - 2i}{5 - 4i} \times \frac{5 + 4i}{5 + 4i} = \frac{(3 - 2i)(5 + 4i)}{(5 - 4i)(5 + 4i)}$$
Now we can write complex solutions to quadratic equations. Solve for the roots of each of these simplifying the radical expression as much as possible.

\[ a) \quad 3x^2 - 2x + 5 = 0 \]

\[ b) \quad 4x^2 + 6x + 3 = 0 \]

What does this mean about the x-intercepts of the graphs of these functions?

\[ a) \quad y = 3x^2 - 2x + 5 \quad b) \quad y = 4x^2 + 6x + 3 \]

If the solutions to a quadratic equation are complex, then the quadratic does not have x-intercepts. It will not cross the x-axis.
Roots of a quadratic:
If \( ax^2 + bx + c = 0 \) then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Are the roots of these real?
If they are real, are they rational or irrational?

a) \( y = 3x^2 - x + 5 \)  
b) \( y = 3x^2 + 8x - 5 \)

c) \( y = x^2 - x + 2 \)  
d) \( y = 3x^2 - 8x - 3 \)