Composition of functions

Inverse functions
Today's objectives

• Define composition of functions
• Give examples of composing functions algebraically and by graphing
• Define inverse function
• Practice finding inverse function algebraically and by graphing
Beads and necklaces

• Few years ago I took up beading for fun. I would buy a bag of varied beads and found that I can make 14 necklaces from it.

• As my beading skills got better, I found that people liked my designs and are willing to pay for my necklaces. I started selling them at a local farmers' market for $9.50.

• I would like to know how much money I will make based on the number of bags of beads I buy.
Definition

Let \( f: A \to B, \ g: B \to C \) be two functions. Composition of \( f \) and \( g \) is a function, denoted by \( g \circ f \), defined by:

\[
g \circ f(x) = g(f(x))
\]
Find \( g \circ f \) if

\[
f(x) = 7x - 2
\]
\[
g(x) = x^3 - 2x
\]

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The following functions can be written as $g \circ f$. What are $f$ and $g$?

$$F(x) = \sqrt{x^2 - 2x + 1}$$

$$F(x) = \frac{x+2}{x+7}$$
Graphing composition of functions
If we did a whole bunch of points
Remember my beading problem?

• As my beading skills got better, I found that people liked my designs and are willing to pay for my necklaces. I started selling them at a local farmers' market for $9.50.

• I would like to know how many necklaces I need to make in order to earn $779.
Interesting question

• If I have a function $f$ can I find function $g$ so that $g \circ f(x) = x$?
Inverse function

• If a function \( f: A \rightarrow B \) has the property that each element of \( B \) is the image of exactly one element of \( A \) (we say \( f \) is injective), then \( f \) has an inverse function, \( f^{-1} \)

\[
\begin{align*}
f \circ f^{-1}(x) &= x \\
f^{-1} \circ f(x) &= x
\end{align*}
\]

• Horizontal line test: Function \( f \) has an inverse if each horizontal line intersects the graph of \( f \) in exactly one point.
Finding the inverse function
Finding expression for inverse

\[ f(x) = 2x + 1 \]

\[ g(x) = \frac{x - 3}{2x + 1} \]

\[ h(x) = 2x^2 + 1 \]
Finding the graph of inverse function
Inverse of \( f \)