Today's lesson and objectives

Graphing and analyzing functions

- Find the zeros of a function
- Determine intervals for which the function is increasing or decreasing
- Determine relative maximum and minimum values of a function
- Use vertical stretch (shrink) to graph a function
- Use vertical and horizontal shifts to sketch a function
- Use vertical and horizontal reflections to graph a function

Intercepts, zeros

- $(0,a)$ is called y-intercept of f if $f(0)=a$.

  Find y-intercept of $g(x)=\sqrt{4-x}$

- $(a,0)$ is called x-intercept of f if $f(a)=0$. In this case, *a* is also called a *zero* of the function f.

  Find x-intercept of $g(x)=\sqrt{4-x}$

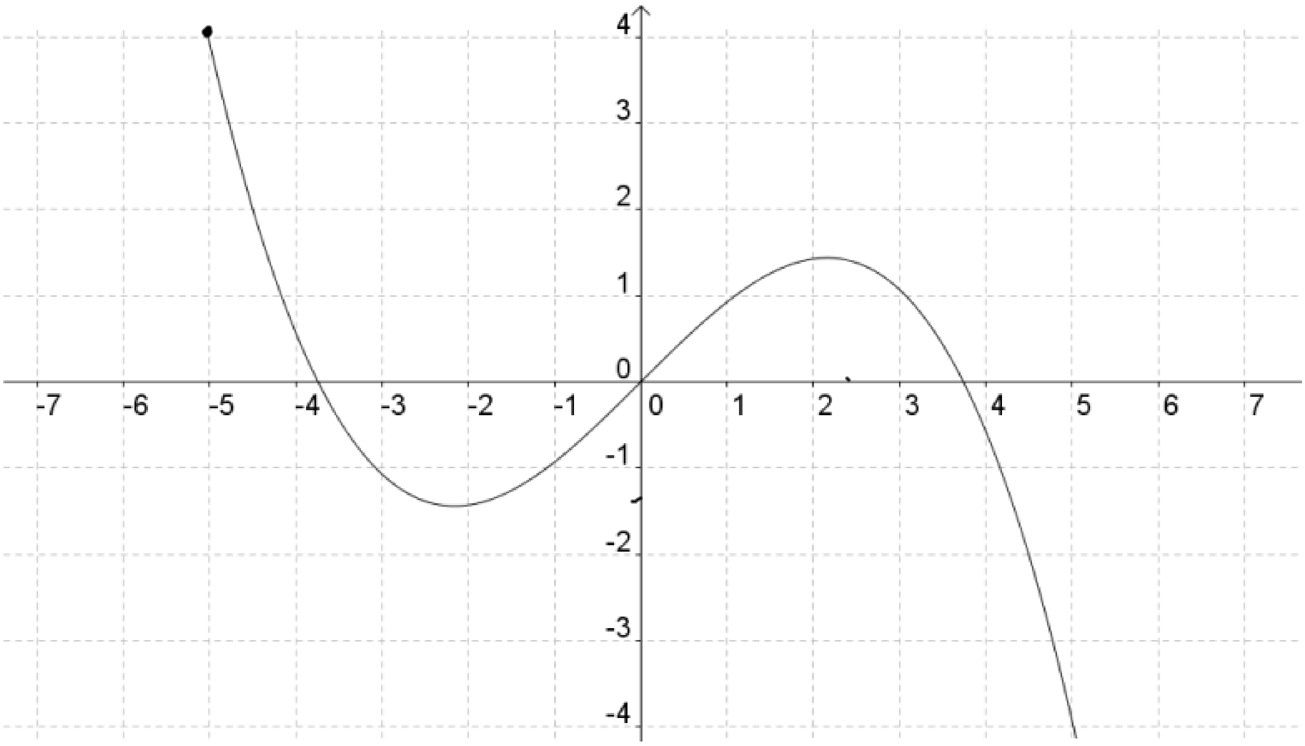
Find x and y intercepts

$$h(x) = |x - 2| - 2$$

Increasing and decreasing functions

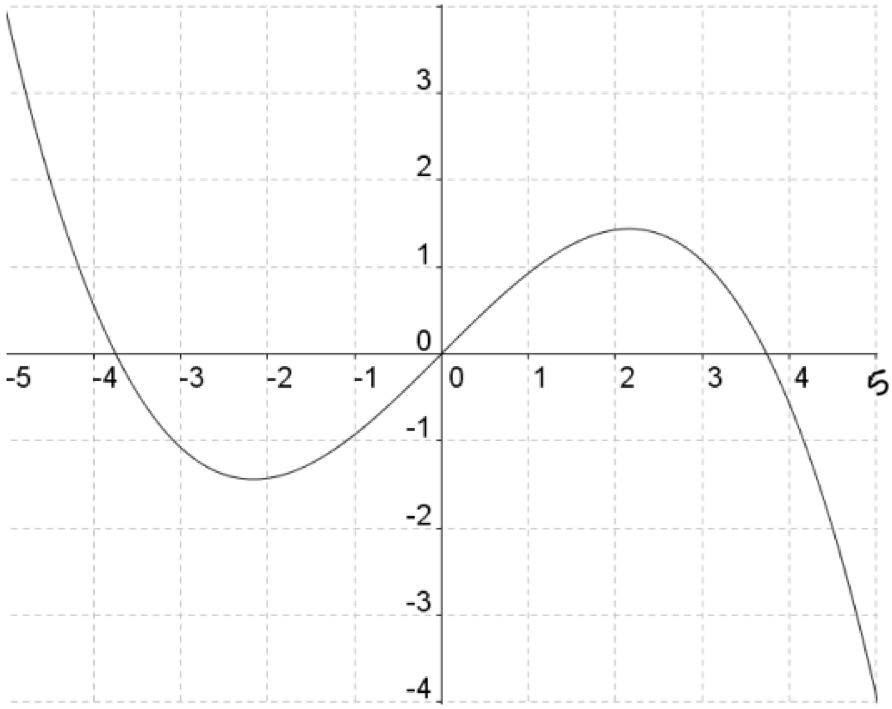
- A function f is **increasing** on an interval if for any two points a and b in the interval for which $a<b$ we have that $f(a)<f(b)$.

- A function f is **decreasing** on an interval if for any two points a and b in the interval for which $a<b$ we have that $f(a)<f(b)$.



Minimum and maximum

- We say that the function f has a **relative (local) minimum** at a point a if $f(a) \leq f(x)$ for all x in some open interval around a.
- We say that the function f has an **absolute (global) minimum** at a point a if $f(a) \leq f(x)$ for all x in the domain.
- We say that the function f has a **relative (local) maximum** at a point a if $f(x) \leq f(a)$ for all x in some open interval around a.
- We say that the function f has an **absolute (global) maximum** at a point a if $f(x) \leq f(a)$ for all x in the domain.



How are these different?

$f(x)=x^2$
$h(x)=x^2-3$
$g(x)=x^2+3$
$k(x)=(x-3)^2$
$l(x)=(x+3)^2$
$m(x)=-(x^2)$

x	$f(x)$	$h(x)$	$g(x)$	$k(x)$	$l(x)$	$m(x)$
-5						
-4						
-3						
-2						
-1						
0						
1						
2						
3						
4						

http://www.coolmath.com/graphit

http://www.geogebra.org

$f(x)=x^2$
$h(x)=x^2-3$
$g(x)=x^2+3$
$k(x)=(x-3)^2$
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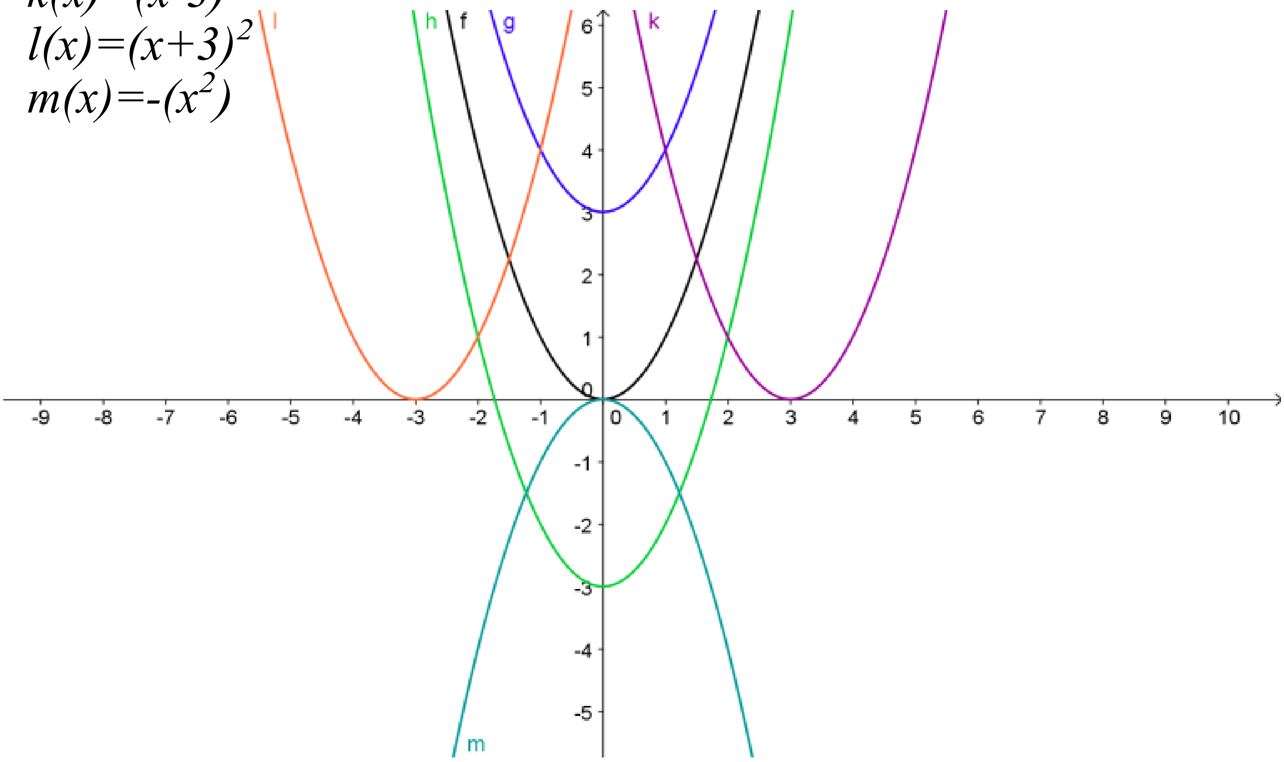


Graph transformations

- If c is any positive number and $f(x)$ any function then:

- The graph of $h(x)=f(x)+c$ is that of f shifted c units upward
- The graph of $g(x)=f(x)-c$ is that of f shifted c units downward
- The graph of $k(x)=f(x-c)$ is that of f shifted c units to the right
- The graph of $l(x)=f(x+c)$ is that of f shifted c units to the left
- The graph of $m(x)=-f(x)$ is that of f reflected along x-axis.

Non-rigid transformations

$f(x)=x^2$
$h(x)=3x^2-2$
$g(x)=3(x^2-2)$
$k(x)=3(x-2)^2$

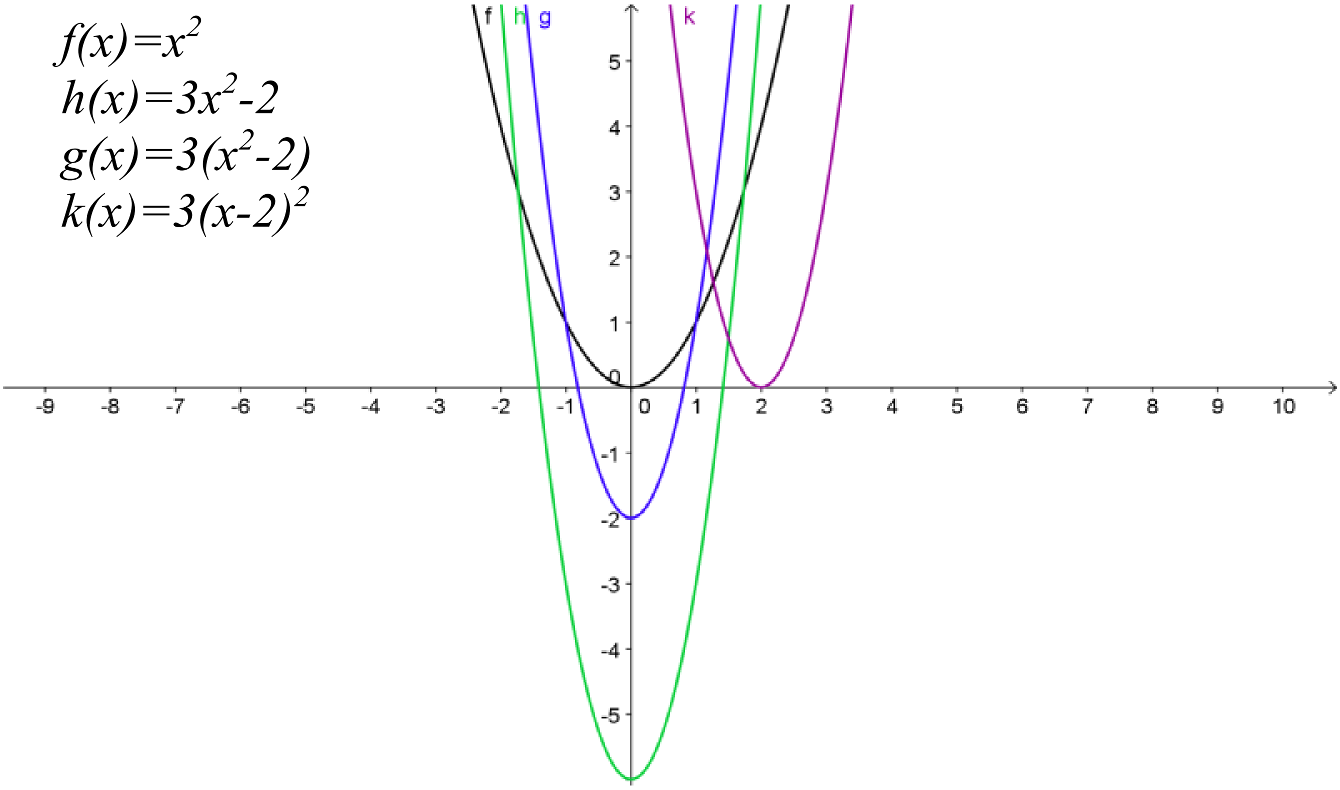
x	$f(x)$	$h(x)$	$g(x)$	$k(x)$
-5				
-4				
-3				
-2				
-1				
0				
1				
2				
3				
4				

$f(x)=x^2$

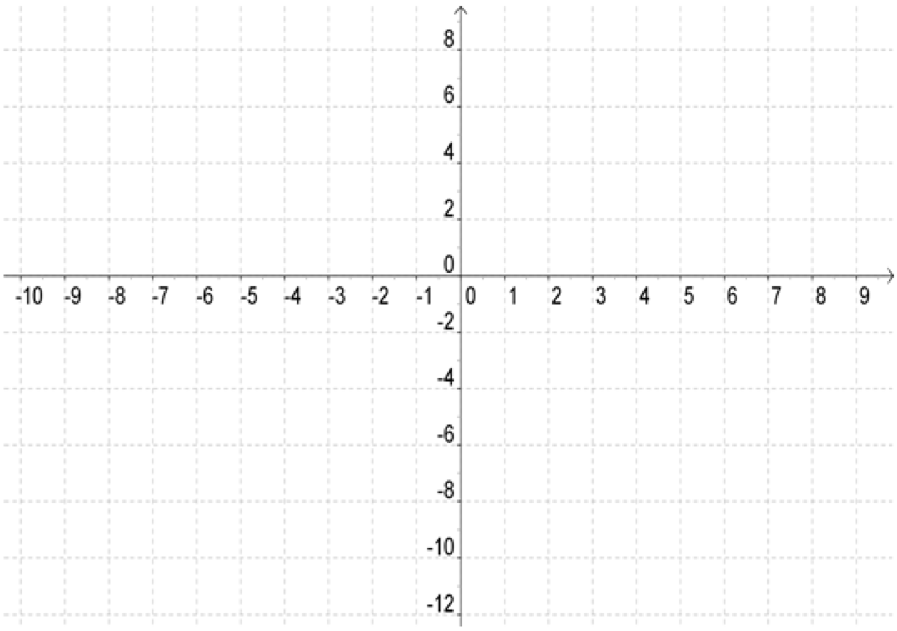
$h(x)=3x^2-2$

$g(x)=3(x^2-2)$

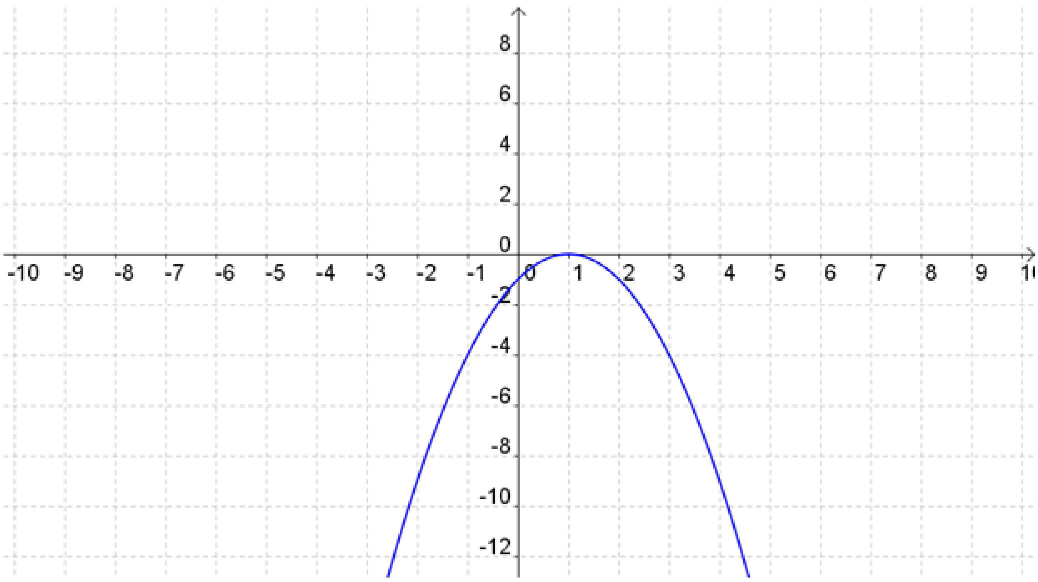
$k(x)=3(x-2)^2$



Graph $f(x) = -(x-1)^2$



8
6
4
2
0
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9
-2
-4
-6
-8
-10
-12



8
6
4
2
0
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9
-2
-4
-6
-8
-10
-12

Library of parent functions

- Linear $f(x)=ax+b$
- Constant $f(x)=c$
- Identity $f(x)=x$
- Quadratic $f(x)=x^2$
- Square root $f(x)=\sqrt{x}$
- Cubic $f(x)=x^3$
- Absolute value $f(x)=|x|$
- Reciprocal $f(x)=1/x$

Draw graphs of each of these functions using symmetries, intercepts, and table of values you learned. Then check your solutions using one of the graphing tools. You should be familiar with all these and be able to recognize the algebraic form upon seeing the graph, as well as know the general form of the graph after seeing the algebraic expression for the function.