Graphing and analyzing functions
Intercepts, zeros

• \((0,a)\) is called y-intercept of \(f\) if \(f(0) = a\). Find y-intercept of \(g(x) = \sqrt{4-x}\)

\[
g(0) = \sqrt{4-0} = \sqrt{4} = 2
\]

\((0, 2)\)

• \((a,0)\) is called x-intercept of \(f\) if \(f(a) = 0\). In this case, \(a\) is also called a zero of the function \(f\).

Find x-intercept of \(g(x) = \sqrt{4-x}\)

\[
\sqrt{4-x} = 0
\]

\[
4-x = 0
\]

\(x = 4\)

\((4, 0)\)
Find x and y intercepts

\[ h(x) = |x - 2| - 2 \]

**x-intercept:** \( h(x) = 0 \)

1) \( |x - 2| - 2 = 0 \)
   \[ |x - 2| = 2 \]
   1) \( x - 2 = 2 \)  \( /+2 \) \( x = 4 \)
   2) \( x - 2 = -2 \)  \( /+2 \) \( x = 0 \)

\( (4, 0) \) \( (0, 0) \) \( y \)-int.

**y-intercept:** \( h(0) = |0 - 2| - 2 \)
   \[ = |2| - 2 = 2 - 2 = 0 \]
Increasing and decreasing functions

- A function $f$ is **increasing** on an interval if for any two points $a$ and $b$ in the interval for which $a < b$ we have that $f(a) < f(b)$.

- A function $f$ is **decreasing** on an interval if for any two points $a$ and $b$ in the interval for which $a < b$ we have that $f(a) < f(b)$. 
Minimum and maximum

• We say that the function $f$ has a **relative (local) minimum** at a point $a$ if $f(a) \leq f(x)$ for all $x$ in some open interval around $a$.
• We say that the function $f$ has an **absolute (global) minimum** at a point $a$ if $f(a) \leq f(x)$ for all $x$ in the domain.
• We say that the function $f$ has a **relative (local) maximum** at a point $a$ if $f(x) \leq f(a)$ for all $x$ in some open interval around $a$.
• We say that the function $f$ has an **absolute (global) maximum** at a point $a$ if $f(x) \leq f(a)$ for all $x$ in the domain.
How are these different?

<table>
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<tr>
<th>x</th>
<th>f(x)</th>
<th>h(x)</th>
<th>g(x)</th>
<th>k(x)</th>
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http://www.coolmath.com/graphit

http://www.geogebra.org
\[ f(x) = x^2 \]
\[ h(x) = x^2 - 3 = f(x) - 3 \]
\[ g(x) = x^2 + 3 = f(x) + 3 \]
\[ k(x) = (x-3)^2 = f(x-3) \]
\[ l(x) = (x+3)^2 = f(x+3) \]
\[ m(x) = -(x^2) = -f(x) \]

\[ k(3) = f(0) \]
Graph transformations

- If $c$ is any positive number and $f(x)$ any function then:

  - The graph of $h(x)=f(x)+c$ is that of $f$ shifted $c$ units upward
  - The graph of $g(x)=f(x)-c$ is that of $f$ shifted $c$ units downward
  - The graph of $k(x)=f(x-c)$ is that of $f$ shifted $c$ units to the right
  - The graph of $l(x)=f(x+c)$ is that of $f$ shifted $c$ units to the left
  - The graph of $m(x)=-f(x)$ is that of $f$ reflected along $x$-axis.
Non-rigid transformations

\[ f(x) = x^2 \]
\[ h(x) = 3x^2 - 2 \]
\[ g(x) = 3(x^2 - 2) \]
\[ k(x) = 3(x - 2)^2 \]

\[ h(x) = 3f(x) - 2 \]
\[ g(x) = 3 \left( f(x) - 2 \right) = 3f(x) - 6 \]
\[ k(x) = 3f(x - 2) \]

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\[ f(x) = x^2 \]
\[ h(x) = 3x^2 - 2 \]
\[ g(x) = 3(x^2 - 2) \]
\[ k(x) = 3(x - 2)^2 \]
Graph \[ f(x) = (x - 1)^2 \]
Library of parent functions

- Linear \( f(x) = ax + b \)
- Constant \( f(x) = c \)
- Identity \( f(x) = x \)
- Quadratic \( f(x) = x^2 \)
- Square root \( f(x) = \sqrt{x} \)
- Cubic \( f(x) = x^3 \)
- Absolute value \( f(x) = |x| \)
- Reciprocal \( f(x) = 1/x \)

Draw graphs of each of these functions using symmetries, intercepts, and table of values you learned. Then check your solutions using one of the graphing tools.