Today's lesson and objectives

Functions

- Identify relations between two variables and determine if they are functions
- Use function notation and evaluate functions
- Determine the domain of a function
- Model problems with functions
Functions

• A function $f$ from set $A$ to set $B$ is a rule that to each element (INDEPENDENT) of the set $A$ assigns EXACTLY one element of the set $B$ (DEPENDENT).

• Set $A$ is called the domain of $f$, while $B$ is called the range of $f$. 
Different ways to describe a function

- Verbally – sentence describing how the dependent and independent variable are related

- Numerically – using a table or list of ordered pairs

- Graphically – drawing all the ordered pairs on a coordinate system (the independent variable corresponds to the horizontal axis, and dependent to vertical)

- Algebraically – writing an expression that describes how one variable depends on the other
Are these functions? Find the domains and ranges.

- There are 120 students in the class M1050.
- To each student in the class M1050 we associate their grade on the final exam.
  
  Domain: 
  Range: 
  Function: yes  no

- To each score 1 to 100 we associate a student with that score.
  
  Domain: 
  Range: 
  Function: yes  no
Are these functions? Find the domains and ranges.

- \{(1,2), (1,3),(2,4),(2,5),(3,6),(3,7)\}
  
  Domain: 
  Range: 
  Function: yes yes

- \{(2,8),(3,7), (4,6),(5,7),(6,8)\}
  
  Domain: 
  Range: 
  Function: yes yes
Is this a function? Find the domain and range.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
<tr>
<td>1</td>
<td>13</td>
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<tr>
<td>2</td>
<td>21</td>
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<tr>
<td>3</td>
<td>17</td>
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<tr>
<td>3</td>
<td>17</td>
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<tr>
<td>4</td>
<td>12</td>
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<td>5</td>
<td>15</td>
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Is this a function? Find the domain and range.

• Is \( y \) a function of \( x \) if we have \( 3x + 5y = 2 \)

• Question: “Do we have only one \( y \) for each \( x \)?

To find that out we should express \( y \) in terms of \( x \), and see if we get a unique (only one) value of \( y \) for each individual \( x \):
Is this a function? Find the domain and range.

• Is $x$ a function of $y$? We have $3x + 5y = 2$

• Question: “Do we have only one $x$ for each $y$?”

To find that out we should express $x$ in terms of $y$, and see if we get a unique (only one) value of $x$ for each individual $y$: 
Function notation and evaluating functions

\[ g(x) = 2x + 4 \]

<table>
<thead>
<tr>
<th>Evaluate function ( g ) at 2, 4, -3, 1/2</th>
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Piecewise defined functions

\[ f(x) = \begin{cases} 
  x^2 - 1, & x \leq 3 \\
  x + 3, & x > 3
\end{cases} \]

- Evaluate \( f \) at 6, -12 and 0
- Draw a table of values for \( x \in [-1, 5] \)
\[ f(x) = \begin{cases} x^2 - 1, & x \leq 3 \\ \end{cases} \]

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
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<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
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</table>
Find the domains of the following functions

\[ g(x) = \sqrt{1 - 2x} \]
Find the domains of the following functions

\[ h(s) = \frac{s(s + 3)}{(s - 2)(s + 4)} \]
Find the domains of the following functions

\[ h(x) = \sqrt[3]{1 - 2x} \]
Graph of a function $f$ is the set of all points $(x, f(x))$ in the coordinate plane.

- Graph $f(x) = 2x - 1$
What can the graph tell us?

• Can I read the value of a function at a given point?

• If I know the value of the function, can I find its origin (the value of independent variable this value corresponds to)?

• Can I read the domain and range?
What can the graph tell us?

\[ f(1) = \]
\[ f(0) = \]
\[ f(-1) = \]

For what \( x \) is
\[ f(x) = 6 \]
\[ f(x) = -4 \]
\[ f(x) = 4 \]
Is this a function? Find its domain and range
Vertical line test

- A curve in the plane is a graph of a function of $x$ only if every vertical line intersects that curve in at most one point.
Review

- Let the function $f$ be defined by $f(x) = \frac{1}{\sqrt{1-x^2}}$

- Indicate whether the following statements are true or false:

  1. $f(x)$ is never positive.
  2. $f(x)$ is never zero.
  3. 0 is in the domain of $f$
  4. All negative real numbers are in the domain of $f$
  5. All positive real numbers are in the domain of $f$
  6. 1 is in the domain of $f$
  7. $f$ is never negative.

http://matti.usu.edu/grapher/