1.2 Graphs of Equations

Vocabulary:

1) Equation in two variables: \( y = 7 - 3x \)

   a) Solution (of equation in two variables)

   \[(1,4): 4 = 7 - 3 \cdot 1\]

   Solution  \( 4 = 4 \)

   b) Graph

   \[
   \begin{array}{c|c|c|c|c|c|c}
   x & -2 & -1 & 0 & 1 & 2 & 3 \\
   \hline
   y & 13 & 10 & 7 & 4 & 1 & -2 \\
   \hline
   y & (-2,13), (-1,10), 
   \end{array}
   \]

c) x-intercept

   \[
   y = 0 \]

   \[
   0 = 7 - 3x \\
   3x = 7 \\
   x = \frac{7}{3}
   \]

d) y-intercept

   \[
   x = 0 \]

   \[
   y = 7 - 3 \cdot 0 \\
   y = 7
   \]

   \:\text{(0,7)}
Examples:

1) Does the point (1,5) lie on the graph of \( y = 4 - |x - 2| \)?

\[
\begin{align*}
5 &< 4 - |1 - 2| \\
5 &< 4 - 1 \\
5 &< 4 \\
5 &\neq 3
\end{align*}
\]

(1,5) does not satisfy the equation and does not lie on this graph.

2) Complete the table of values and sketch the graph of \( y = 5 - x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
2) Symmetry:

a) With respect to the x-axis

\((x, y)\) and \((-x, -y)\) lie on the graph.

b) With respect to the y-axis

\((x, y)\) and \((-x, y)\) lie on the graph.

c) With respect to the origin

\((x, y)\) and \((-x, -y)\)
3) Find the x-intercept and the y-intercept and use symmetry to sketch a graph of these equations.

\[ y^2 = x + 1 \]

**x-intercept**

\[ y = 0 \quad 0 = x + 1 \]

\[ x = -1 \]

\( (-1, 0) \)

**y-intercept**

\[ x = 0 \]

\[ y^2 = 1 \]

\[ y = \pm 1 \]

\( (0, 1), (0, -1) \)

Symmetry:

\[ (-y)^2 = y^2 = x + 1 \]

\[ x \quad 3 \quad 8 \]

\[ y \quad 2 \quad 3 \]
4) Find the x-intercept and the y-intercept and use symmetry to sketch a graph of these equations.

\[ y = x^4 - x^2 + 3 \]

**x-intercept**

\[ y = 0 \]

\[ 0 = x^4 - x^2 + 3 \]

\[ 0 = 1 \cdot (x^2)^2 - (x^2) + 3 \]

\[ x^2 = \frac{1 \pm \sqrt{1 - 4 \cdot 3}}{2} = \frac{1 \pm \sqrt{-11}}{2} \]

**there is no x-int.**

**y-intercept**

\[ x = 0 \]

\[ y = 0 - 0 + 3 = 3 \]  \[ (0, 3) \]

**symmetry**

\[ y = (-x)^4 - (-x)^2 + 3 \]

\[ y = x^4 - x^2 + 3 \]

symmetric w/ respect to y axis
4) Find the x-intercept and the y-intercept and use symmetry to sketch a graph of these equations.

\[ xy = 4 \]

no intercepts

symmetry

\[-x)(-y) = xy = 4\]

symmetry about
the origin

<table>
<thead>
<tr>
<th>x</th>
<th>1/4</th>
<th>1/2</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>10</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>1/2</td>
</tr>
</tbody>
</table>
What is the standard equation of the circle with center at (-3,2) with radius 3:

\[ q = (x - (-3))^2 + (y - 2)^2 = (x + 3)^2 + (y - 2)^2 \]
5) Write the standard form of the equation of a circle with endpoints of the diameter at (-4,-1) and (4,1)

\[ M \left( \frac{4+(-4)}{2}, \frac{1+(-1)}{2} \right) = \left( \frac{0}{2}, \frac{0}{2} \right) = (0,0) \]

\[ r = d = \sqrt{(4-0)^2 + (1-0)^2} = \sqrt{4^2 + 1^2} = \sqrt{17} \]

\[
\left( x - 0 \right)^2 + \left( y - 0 \right)^2 = 17 \\
\therefore x^2 + y^2 = 17
\]