Math 1030 #9a

Savings Plans and Investments

Savings Plan Formulas
\[ A = P(1 + APR)^Y \]

**Compound interest formula (compounding annually)**

**Savings Plan Formula** (regular payments)

Suppose you invest $1000 at the end of each year for 5 years in an account that pays 10% interest compounded annually. What is the value after 5 years (future value)?

<table>
<thead>
<tr>
<th>Year</th>
<th>amount generated</th>
<th>$1000 earns 4 yrs of interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000(1+0.1)^4</td>
<td>$1464.10</td>
</tr>
<tr>
<td>2</td>
<td>1000(1+0.1)^3</td>
<td>$1331.00</td>
</tr>
<tr>
<td>3</td>
<td>1000(1+0.1)^2</td>
<td>$1210.00</td>
</tr>
<tr>
<td>4</td>
<td>1000(1+0.1)^1</td>
<td>$1100.00</td>
</tr>
<tr>
<td>5</td>
<td>1000(1+0.1)^0</td>
<td>$1000.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1000(1+0.1)^0</td>
<td><strong>$6105.10</strong></td>
</tr>
</tbody>
</table>

\[ A = \text{PMT} \cdot \frac{(1 + \frac{APR}{n})^{nY} - 1}{\frac{APR}{n}} \]

**Savings plan formula**

(Where you deposit/invest same amt of $ every period and want to know future value)

(Also called future value of an ordinary annuity)
EX 1: Find the savings plan balance after 5 years with an APR of 2.5% with monthly payments of $100.

\[ A = PMT \cdot \frac{(1 + \frac{APR}{n})^{nY} - 1}{\frac{APR}{n}} \]

\( A \) = balance after \( Y \) years
\( APR \) = annual interest rate
\( n \) = number of payment periods per year
\( Y \) = number of years
\( PMT \) = regular payment amount

(hidden assumption: compounding occurs monthly, i.e. payment periods match compounding period)

\( Y = 5 \), \( APR = 0.025 \), \( PMT = 100 \), \( n = 12 \)

\[ A = 100 \left[ \frac{(1 + \frac{0.025}{12})^{12(5)} - 1}{\frac{0.025}{12}} \right] \]

\[ = 100 \left[ \frac{(1.0020833)^{60} - 1}{0.0020833} \right] \]

\[ \approx \$6,384.05 \]

Note: we put in \( 100(12)(5) = \$6000 \).
EX 2: At age 28 you begin saving $50 at the end of each month in an account with an APR of 4%. How much will the balance be when you retire at age 65? How does this compare to the amount invested?

\[ A = PMT \cdot \frac{(1 + \frac{APR}{n})^n - 1}{\frac{APR}{n}} \]

\[ \text{PMT} = 50, \ n = 12, \ APR = 0.04 \]

\[ y = 65 - 28 = 37 \]

\[ A = 50 \left[ \frac{(1 + \frac{0.04}{12})^{12(37)}}{0.04} - 1 \right] \]

\[ \approx 50,732.21 \]

Compare w/ total amt deposited: \(50(12)(37) = 22,200.00\)

EX 3: At age 23 when you graduate, you start saving for retirement. Your investment plan pays an APR of 4.5%. You want to have $5 million when you retire in 45 years. How much should you deposit monthly?

\[ A = PMT \cdot \frac{(1 + \frac{APR}{n})^n - 1}{\frac{APR}{n}} \]

\[ \text{APR} = 0.045, \ A = 5,000,000 \]

\[ y = 45, \ n = 12 \]

\[ 5,000,000 = \text{PMT} \left[ \frac{(1 + \frac{0.045}{12})^{12(45)}}{0.045} - 1 \right] \]

\[ \frac{5,000,000}{0.045} = \text{PMT} \left[ 1.00375^{540} - 1 \right] \]

\[ 5,000,000 = (1745.992368) \text{PMT} \]

\[ \$2,863.70 = \text{PMT} \]

\[ 2863.70 \times (12)(45) = \$1,546,398 \]