Math 1030 #17c
Fundamentals of Geometry
3-D Geometry
A Few Formulas

**Surface area**

(2-d measurement)
the amount of 2-d space it takes to cover outside of surface

- **Sphere**: 
  \[ S = 4\pi r^2 \]
  units: \( \text{cm}^2, \text{m}^2, \text{ft}^2, \text{in}^2, \text{cm}^2, \text{m}^2, \text{etc.} \)

- **Cube**: 
  \[ S = 6x^2 \]

- **Rectangular box** (prism): 
  \[ S = 2lw + 2wh + 2lh \]
  \( \text{front \ top \ sides \ back \ bottom} \)

- **Right circular cylinder**: 
  \[ S = 2\pi r^2 + 2\pi rh \]
  \( \text{top \ + \ bottom \ lateral \ area} \)

(3-d measurement)
the amount of 3-d space inside the surface

- **Sphere**: 
  \[ V = \frac{4}{3}\pi r^3 \]
  units: \( \text{cm}^3, \text{m}^3, \text{ft}^3, \text{in}^3, \) etc.

- **Cube**: 
  \[ V = (x\times x) = x^3 \]
  \( \text{area of base} \ (x) \ \text{height} \)

- **Rectangular box**: 
  \[ V = (lw)h = lwh \]
  \( \text{area of base} \ \text{height} \)

- **Cylinder**: 
  \[ V = (\pi r^2)h = \pi r^2h \]
  \( \text{area of base} \ \text{height} \)
EX 1: A warehouse sells cylindrical tanks in these dimensions:
1. radius 40 ft and height 80 ft
2. radius 5 ft and height 8 ft.

a) How many of the smaller tanks would you need to purchase to hold the same amount as one of the larger tanks?

Measuring surface area or volume?
- We want space inside tanks so we want volume.

\[ V_L = \pi r^2 h = \pi (40 \text{ ft})^2 (80 \text{ ft}) = 40^2 \cdot 80\pi \text{ ft}^3 = 128,000\pi \text{ ft}^3 \]

\[ V_{Sm} = \pi r^2 h = \pi (5 \text{ ft})^2 (8 \text{ ft}) = 25\pi \cdot 8 \text{ ft}^3 = 200\pi \text{ ft}^3 \]

\[ \frac{128,000\pi \text{ ft}^3}{200\pi \text{ ft}^3} = \frac{1280}{2} = 640 \]

⇒ The large tank holds the same amount as 640 small tanks.

b) Compare the surface area of the larger tank to the total surface area of all the smaller tanks.

(assume all tanks have their lids)

\[ S_L = 2\pi r^2 + 2\pi rh = 2\pi (r^2 + rh) \]
\[ = 2\pi (40^2 + (40)(80)) \]
\[ = 9600\pi \text{ ft}^2 \]

\[ S_{Sm} = 2\pi (r^2 + rh) \]
\[ = 2\pi ((5^2) + (5)(8)) \]
\[ = 130\pi \text{ ft}^2 \]

640 \[ S_{Sm} = 640(130\pi)\text{ ft}^2 = 83,200\pi \text{ ft}^2 \]

\[ \frac{83,200\pi \text{ ft}^2}{9600\pi \text{ ft}^2} = \frac{8}{3} \]

⇒ Surface area of the 640 smaller tanks is \( \frac{8}{3} \) times as much as surface area of the one large tank.
EX 2: I want to fill this box with skittles for a gift.

a) How much paper will it take to just cover the box (in square inches)?

\[ S = 2(8 \cdot 5) + 2(3 \cdot 5) + 2(8 \cdot 3) \]
\[ = 158 \text{ in}^2 \]

b) How many skittles will it hold, assuming there are 32 skittles per cubic inch?

\[ V = (3 \cdot 8 \cdot 5) = 120 \text{ in}^3 \]
\[ \frac{32 \text{ skittles}}{\text{in}^3} \cdot 120 \text{ in}^3 = 3840 \text{ skittles} \]

c) If I want to tape the box in all three directions, how much tape will it take?

\[ L = (3 + 8 + 3 + 8) + (5 + 8 + 5 + 8) + (3 + 5 + 3 + 5) \]
\[ = 4(3) + 4(5) + 4(8) \]
\[ = 64 \text{ in} \]