Math 1030 #16b

Exponential Modeling

Graphing Exponential Functions

exponential growth

doubling time

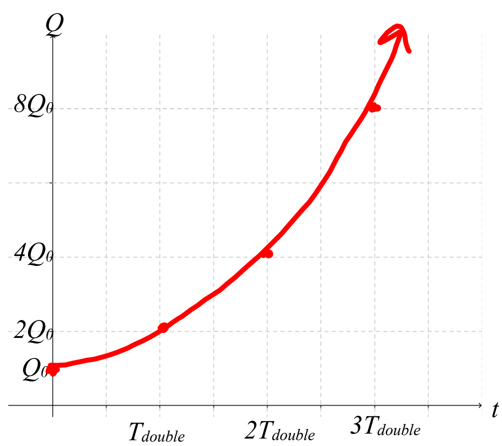
exponential decay

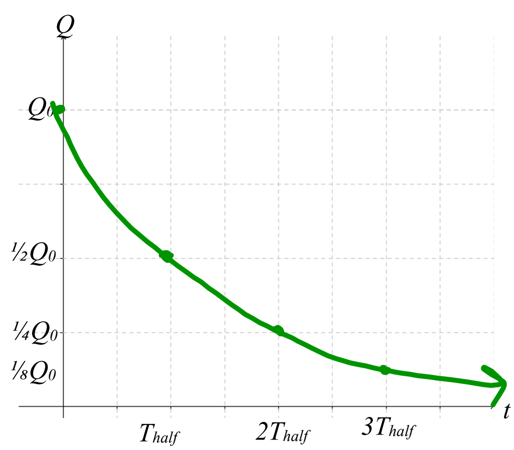
half-life

The easiest way to graph exponential functions is to use points corresponding to several doubling times (or half-lives in the case of decay).

For growth:
- Start at $(0, Q_0)$
- Plot $(T_{double}, 2Q_0)$, $(2T_{double}, 4Q_0)$, $(3T_{double}, 8Q_0)$, etc.

For decay:
- Start at $(0, Q_0)$
- Plot $(T_{half}, \tfrac{1}{2}Q_0)$, $(2T_{half}, \tfrac{1}{4}Q_0)$, $(3T_{half}, \tfrac{1}{8}Q_0)$, etc.





$$T_{double} = \frac{\log_{10} 2}{\log_{10}(1+r)}$$

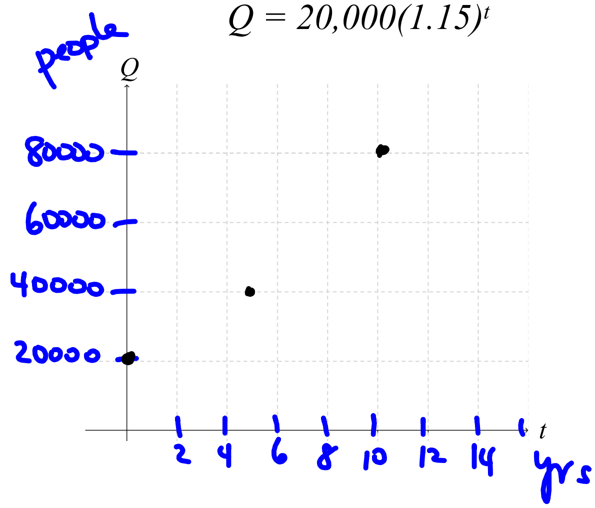
$$r > 0$$

$$T_{half} = -\frac{\log_{10} 2}{\log_{10}(1+r)}$$

$$r < 0$$

EX 1: Graph the following equations from the previous lesson.

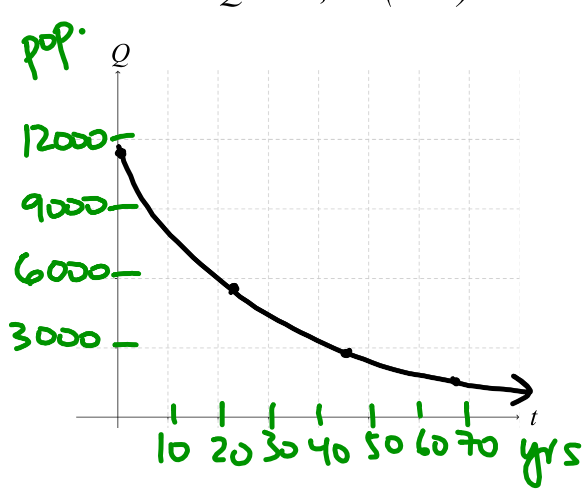
a) The growth of the population of Heber, Utah is
$Q = 20,000(1.15)^t$



$T_{double} = \frac{\log 2}{\log(1.15)} \approx 4.96$

≈ 5 yrs

b) The decline of the population of Cook Islands is
$Q = 11,000(0.97)^t$



$T_{half} = \frac{-\log 2}{\log 0.97}$

$\simeq 22.75 \cong 23$ yrs

Alternate Forms of the Exponential Function

(1) $Q = Q_0(1+r)^t$ Note: r is positive for growth and negative for decay.

(2A) $Q = Q_0(2)^{t/T_{double}}$ for growth

(2B) $Q = Q_0(1/2)^{t/T_{half}}$ for decay

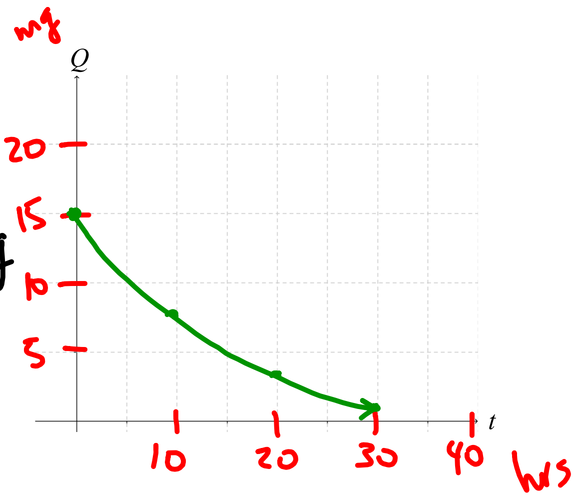
- if you are given r, use eqn (1)
- if you're given T_{half} or T_{double} use one of eqn (2)

EX 2: The half-life of a certain antibiotic in the bloodstream is 10 hours. If you are given a 15 mg shot of this antibiotic at midnight, write an equation for and sketch a graph showing the amount in your bloodstream for the next 24 hours.

because we are given $T_h = 10$ hrs, use (2B)

$$Q = Q_0\left(\tfrac{1}{2}\right)^{\frac{t}{10}} \qquad Q_0 = 15\text{mg}$$

$$\boxed{Q = 15\left(\tfrac{1}{2}\right)^{t/10}}$$



note: eqn (2B) $\rightarrow$ eqn (1)

$$\left(\tfrac{1}{2}\right)^{\frac{t}{10}} = \left[\left(\tfrac{1}{2}\right)^{\frac{1}{10}}\right]^t \simeq 0.933^t$$

$$\Rightarrow \quad Q = 15\left(\tfrac{1}{2}\right)^{\frac{t}{10}} \simeq 15(0.933)^t \quad \text{of form (1)}$$

$0.933 = 1 + r \quad \Rightarrow r \simeq -6.7\%$