Math 1030 #16b

Exponential Modeling

Graphing Exponential Functions
The easiest way to graph exponential functions is to use points corresponding to several doubling times (or half-lives in the case of decay).

For growth:
Start at \((0, Q_0)\)
Plot \((T_{\text{double}}, 2Q_0)\),
\((2T_{\text{double}}, 4Q_0)\),
\((3T_{\text{double}}, 8Q_0)\), etc.

For decay:
Start at \((0, Q_0)\)
Plot \((T_{\text{half}}, \frac{1}{2}Q_0)\),
\((2T_{\text{half}}, \frac{1}{4}Q_0)\),
\((3T_{\text{half}}, \frac{1}{8}Q_0)\), etc.

\[
T_{\text{double}} = \frac{\log_{10}2}{\log_{10}(1+r)}
\]

\[
T_{\text{half}} = -\frac{\log_{10}2}{\log_{10}(1+r)}
\]

\(r > 0\)

\(r < 0\)
EX 1: Graph the following equations from the previous lesson.

a) The growth of the population of Heber, Utah is
   \[ Q = 20,000(1.15)^t \]
   
   Graph:
   - y-axis: Q (people)
   - x-axis: t (years)
   - Points at t=2, 4, 6, 8, 10, 12, 14

   \[ T_{\text{double}} = \frac{\log 2}{\log (1.15)} \approx 4.96 \]
   \[ \approx 5 \text{ yrs} \]

b) The decline of the population of Cook Islands is
   \[ Q = 11,000(0.97)^t \]
   
   Graph:
   - y-axis: Q (pop.)
   - x-axis: t (years)
   - Points at t=10, 20, 30, 40, 50, 60, 70

   \[ T_{\text{half}} = \frac{-\log 2}{\log 0.97} \]
   \[ \approx 22.75 \approx 23 \text{ yrs} \]
Alternate Forms of the Exponential Function

\[ Q = Q_0(1+r)^t \quad \text{Note: } r \text{ is positive for growth and negative for decay.} \]

\[ Q = Q_0(2)^{t/T_{\text{double}}} \quad \text{for growth} \]

\[ Q = Q_0(1/2)^{t/T_{\text{half}}} \quad \text{for decay} \]

EX 2: The half-life of a certain antibiotic in the bloodstream is 10 hours. If you are given a 15 mg shot of this antibiotic at midnight, write an equation for and sketch a graph showing the amount in your bloodstream for the next 24 hours.

Because we are given \( T_h = 10 \text{ hrs, use (2B)} \)

\[ Q = Q_0 \left( \frac{1}{2} \right)^{\frac{t}{10}} \]

\[ Q = 15 \left( \frac{1}{2} \right)^{\frac{t}{10}} \]

\[ Q = 15 \left( \frac{1}{2} \right)^{\frac{t}{10}} \]

Note: eqn (2B) \( \Rightarrow \) eqn (1)

\[ \left( \frac{1}{2} \right)^{t/10} = \left( \left( \frac{1}{2} \right)^{1/10} \right)^t \approx 0.933^t \]

\[ \Rightarrow Q = 15 \left( \frac{1}{2} \right)^{t/10} \approx 15 \left( 0.933 \right)^t \quad \text{of form (1)} \]

\[ 0.933 = 1+r \quad \Rightarrow r \approx 6.7\% \]