Math 1030 #12b
Doubling Time and Half-Life
Half-Life
**Half-life** is the time it takes for half of a population to vanish if it decreases by the same percent each time period. 

EX 1: If a city of 1000 people is decreasing by 10% each year, when will there be half as many people?

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of people</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td><strong>1000</strong></td>
</tr>
<tr>
<td>1</td>
<td>$0.9(1000) = 900$</td>
</tr>
<tr>
<td>2</td>
<td>$0.9^2(1000) = 810$</td>
</tr>
<tr>
<td>3</td>
<td>$0.9(0.9^2)(1000) = 0.9^3(1000) = 729$</td>
</tr>
<tr>
<td>4</td>
<td>$0.9^4(1000) = 656$</td>
</tr>
<tr>
<td>5</td>
<td>$0.9^5(1000) = 595$</td>
</tr>
<tr>
<td>6</td>
<td>$0.9^6(1000) = 531$</td>
</tr>
<tr>
<td>7</td>
<td>$0.9^7(1000) = 478$</td>
</tr>
</tbody>
</table>

=> **Half-life is about 7 yrs**
After a time, $t$, an exponentially decaying quantity with a half life of $T_{\text{half}}$ decreases in size by a factor of $(1/2)^{t/T}$. The new value is related to the initial value by $\text{new value} = \text{initial value} \times (1/2)^{t/T}$.

**Approximate Half-life**

For a quantity decaying exponentially at a rate of $P\%$ per time period

$$T_{\text{half}} \approx \frac{70}{P}$$

This works best for small rates and breaks down for rates over about $15\%$. 
EX 2: Radioactive carbon-14 has a half-life of about 5700 years. It collects in organisms only while they are alive. Once they are dead, it only decays. What fraction of carbon-14 in an animal bone still remains 800 years after the animal has died?

\[ T_{\text{half}} = 5700 \text{ yrs} \quad t = 800 \text{ yrs} \]

decreases (shrinks) by factor

\[ \left( \frac{1}{2} \right)^{\frac{800}{5700}} \approx 0.9073 \]

\[ \Rightarrow \text{about 91\% of carbon-14 still remains after 800 yrs} \]

EX 3: A clean-up project is reducing the concentration of a pollutant in the water supply, with a 3.5% decrease per week.

a) What is the approximate half-life of the concentration of the pollutant?

\[ T \approx \frac{70}{3.5} = 20 \text{ weeks} \]

b) What fraction of the original will remain after one year?

\[ 1 \text{ yr} = 52 \text{ weeks}, \ t = 52 \text{ wks}, \ T_h = 20 \text{ wks} \]

decay (shrink) factor = \( \left( \frac{1}{2} \right)^{\frac{52}{20}} \)

\[ = \left( \frac{1}{2} \right)^{2.6} \approx 0.165 \]

\[ \Rightarrow \text{about 16.5\% of original pollutants remain after 1 yr} \]
Exact half-life formula:

$$T_{\text{half}} = -\frac{\log_{10}(2)}{\log_{10}(1+r)}$$

where \( r \) is a decimal and negative.

Note: The units of time for \( r \) and \( T \) must be the same (per month, year, etc.)

EX 4: Suppose the Russian ruble is falling in value against the dollar at 11% per year. \( r = 0.11, \quad P = 11 \)

a) Approximately how long will it take the ruble to lose half its value?

$$T_h \approx \frac{70}{11} \approx 6.36 \text{ yrs}$$

b) Exactly how long will it take the ruble to lose half its value?

$$T_h = -\frac{\log 2}{\log (1+0.11)} \approx -\frac{\log 2}{\log (0.89)} \approx 5.948$$