Math 1030 #12a

Exponential Decay

Exponential Growth

Doubling Time and Half-Life

Doubling Time

Half-life

Doubling

Exponential growth leads to repeated doublings.

Exponential decay leads to repeated halving.

EX 1: If you get a salary increase of 10% each year, in what year will your salary be double what it is today?

After a time, t, an exponentially growing quantity with a doubling time of T_{double} increases in size by a factor of $2^{t/T}$. The new value is related to the initial value by

new value = *initial value* × $2^{t/T}$.

EX 2: Suppose your bank account has a doubling time of 11 years. By what factor does your balance increase in 34 years?

EX 3: The initial population of a town is 10,000 and it grows with a doubling time of 8 years. What will the population be in

a) 12 years?

b) 24 years?

It can be interesting to look at the time it takes your money in a bank to double.

EX 4: If you place $1000 in an account that pays 9% annual interest, compounded annually, during what year will it double?

$$A = P(1+APR)^y$$

Year	Amount

Rule of 70

For a quantity growing exponentially at a rate of $P\%$ per time period, the doubling time is approximately

$$T_{double} \approx \frac{70}{P}.$$

This works best for small growth rates and breaks down for growth rates over about 15%.

EX 5: Determine about how many years it will take you to double your money at these annual interest rates.

a) 3% b) 5% c) 8%

EX 6: The world population was about 6.8 billion in 2005 and was growing at a rate of about 1.2% per year.

a) What is the approximate doubling time?

b) If this growth rate continues, what would the population be in 2019?

<u>Exact doubling time formula</u>:

$$T_{double} = \frac{log_{10}(2)}{log_{10}(1+r)}$$ where r is a decimal and positive.

Note: The units of time for r and T must be the same (per month, year, etc.)

EX 7: Oil consumption is increasing at a rate of 2.2% per year.

a) What is the approximate doubling time?

b) What is the exact doubling time?