Math 1030 #10a
Loans, Credit Cards and Mortgages

Loans
Loans, credit cards and mortgages are ways you borrow money. When you borrow money, the bank is basically making a lump sum investment and getting a periodic return.

**Principal** is the amount of money owed at any particular time.

Interest is charged on the principal.

To pay off a loan, you should pay the interest and also some part of the principal.

An **installment loan** (amortized loan) is a loan payed off with equal regular payments.

**Loan Payment Formula** (Installment loans)

\[
PMT = \frac{P \cdot \left(\frac{\text{APR}}{n}\right)}{\left[1 - (1 + \frac{\text{APR}}{n})^{-nY}\right]}
\]

- **PMT** = regular payment amount
- **P** = starting principal
- **APR** = annual percentage rate
- **n** = number of payments per year
- **Y** = the term of the loan (years)
EX 1: You have a student loan of $40,000 with an APR of 6%. Compare monthly payment amounts and total amount paid for these options.

\[
P = 40,000 \\
n = 12 \\
\text{APR} = 0.06
\]

15 years

\[
PMT = \frac{P \cdot \left(\frac{\text{APR}}{n}\right)}{1 - (1 + \frac{\text{APR}}{n})^{-nY}}
\]

\[
Y = 15
\]

\[
PMT = \frac{40000 \cdot \left(\frac{0.06}{12}\right)}{1 - \left(1 + \frac{0.06}{12}\right)^{-12(15)}}
\]

\[
= \frac{40000 \cdot (0.005)}{1 - 1.005^{-180}}
\]

\[
\approx 337.54
\]

total paid (over the 15 yrs) = \$337.54(12)(15) 
\approx \$60,757.20

25 years

\[
Y = 25
\]

\[
PMT = \frac{40000 \cdot (0.005)}{1 - 1.005^{-12(25)}}
\]

\[
\approx 257.72
\]

total paid:

\[
257.72(12)(25)
\]

\[
\approx 77,316.00
\]

EX 2: For the loan in example 1, make a table showing the amounts of each monthly payment that goes to the principal and to the interest for the first three months. ($40,000 at 6% for 15 yrs)

<table>
<thead>
<tr>
<th>Loan balance</th>
<th>Interest</th>
<th>Payment</th>
<th>Principal paid</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 40000</td>
<td>40000(0.005) = 200</td>
<td>337.54</td>
<td>337.54 - 200 = 137.54</td>
</tr>
<tr>
<td>2 (40000 - 137.54) = 39862.46</td>
<td>39862.46(0.005) = 199.31</td>
<td>337.54</td>
<td>337.54 - 199.31 = 138.23</td>
</tr>
<tr>
<td>3 (39862.46 - 138.23) = 39724.23</td>
<td>39724.23(0.005) = 198.62</td>
<td>337.54</td>
<td>337.54 - 198.62 = 138.92</td>
</tr>
</tbody>
</table>
EX 3: You borrow $4000 to buy a used car. You can afford monthly payments of $150. Which of these meets your needs?

1) 2 years at 8% APR

\[ \text{PMT} = \frac{P \cdot \left( \frac{\text{APR}}{n} \right)}{1 - \left( 1 + \frac{\text{APR}}{n} \right)^{-nY}} = \frac{4000 \left( \frac{0.08}{12} \right)}{1 - \left( 1 + \frac{0.08}{12} \right)^{-12(2)}} \approx 180.90 \]

2) 3 years at 9% APR

\[ \text{PMT} = \frac{4000 \left( \frac{0.09}{12} \right)}{1 - \left( 1 + \frac{0.09}{12} \right)^{-12(3)}} \approx 127.19 \]

3) 4 years at 10% APR

\[ \text{PMT} = \frac{4000 \left( \frac{0.10}{12} \right)}{1 - \left( 1 + \frac{0.10}{12} \right)^{-12(4)}} \approx 101.45 \]

EX 4: A payday loan company charges $150 to borrow $1000 for 2 weeks. What is the APR?

$150 is 15% of $1000 (only for 2 weeks)

\[ \frac{15\%}{2\, \text{weeks}} \cdot \frac{52\, \text{weeks}}{1\, \text{yr}} = 15\% \left( \frac{26}{1} \right) = 390\% \, \text{(per year)} \]

\[ \Rightarrow \text{APR} = 390\% \]