Section 9.5: Solving Exponential and Logarithmic Equations

Objectives:

- Solve basic exponential and logarithmic equations.
- Use inverse properties to solve exponential and logarithmic equations.

\[ \log_2(x-2) = \log_2 x + 3 \]

\[ 500e^{-0.2x} = 100 \]
Solve

1) \[ 9^{x+3} = 9^{10} \]
   \[ x + 3 = 10 \]
   \[ x = 7 \]

2) \[ \log_3(4 - 3x) = \log_3(2x + 9) \]
   \[ 4 - 3x = 2x + 9 \]
   \[ 4 = 5x + 9 \]
   \[ -5 = 5x \Rightarrow x = -1 \]

3) \[ 6e^{-x} = 3 \]
   \[ e^{-x} = \frac{1}{2} \]
   \[ x = -\ln\left(\frac{1}{2}\right) = \ln 2 \]

Strategy to solve exp. eqn

1. Isolate exponential term.
2. Use def'n log to rewrite the eqn; or take log of both sides (choose appropriate base for log)
3. Finish solving

Notes:
- exponential eqns - do not need to check answers
- logarithmic eqns - it is necessary to check answers (because we can only take log of a positive #)
4) \[
50(3 - e^{2x}) = 125
\]
\[
\frac{50}{3 - e^{2x}} = \frac{125}{50}
\]
\[
3 - e^{2x} = \frac{5}{2}
\]
\[
e^{2x} = \frac{3}{2} - 3
\]
\[
e \rightarrow e^x = \frac{1}{2}
\]
\[
\ln \left(\frac{1}{2}\right) = \frac{2x}{2}
\]
\[
\frac{1}{2} \ln\left(\frac{1}{2}\right) = x
\]
\[
x = \ln \sqrt{\frac{1}{2}} = -\ln \sqrt{2}
\]

5) \[
\frac{500}{1 + e^{-0.1x}} = 400
\]
\[
500 = 400(1 + e^{-0.1x})
\]
\[
\frac{5}{4} = 1 + e^{-0.1x}
\]
\[
\frac{5}{4} - 1 = e^{-0.1x}
\]
\[
\frac{1}{4} = e^{-0.1x}
\]
\[
\ln \left(\frac{1}{4}\right) = -0.1x
\]
\[
\frac{-1}{0.1} \ln\left(\frac{1}{4}\right) = x = 10 \ln \left(\frac{1}{4}\right)
\]
\[
= 10 \ln 4
\]

6) \[
\frac{2}{3} \log_3(x + 1) = -1
\]
\[
\log_3(x + 1) = -\frac{3}{2}
\]
\[
-\frac{3}{2} \cdot x = x + 1
\]
\[
3 = x + 1
\]
\[
-1
\]
\[
x = \frac{3^{-1/2} - 1}{1} = \sqrt{3^{-1/2}} - 1
\]

Strategy for solving logarithmic eqn:

1. Use log properties to condense log terms completely.
2. Use defn of log to rewrite eqn in exp. form.
3. Finish solving.
4. Check answer.
7) \[ \log_3(x - 2) + \log_3 5 = 3 \]

\[
\begin{align*}
\log_3(5(x-2)) &= 3 \\
3 &= 5(x-2) \\
27 &= 5x - 10 \\
37 &= 5x \\
\end{align*}
\]

\[ x = \frac{37}{5} \quad \text{or} \quad 7.4 \]

8) \[ \log_3(2x) + \log_3(x - 1) - \log_3 4 = 1 \]

\[
\begin{align*}
\log_3\left(\frac{2x(x-1)}{4}\right) &= 1 \\
4 \cdot \frac{2x(x-1)}{4} &= 3 \\
12 &= 2x(x-1) \\
12 &= 2x^2 - 2x \\
0 &= 2x^2 - 2x - 12 \\
0 &= 2(x^2 - x - 6) \\
0 &= 2(x-3)(x+2) \\
0 &= (x-3)(x+2) \\
\end{align*}
\]

\[
\begin{align*}
x - 3 &= 0 & x + 2 &= 0 \\
x &= 3 & x &= -2
\end{align*}
\]

Soln: \( x = 3 \)
Applications

1) At what interest rate (compounded continuously) will you have to invest $10,000 to make sure it doubles in ten years?

\[ P = Pe^{rt} \]
\[ P = 10000 \]
\[ r = ? \]
\[ t = 10 \]
\[ y = 20000 \]

\[ 20000 = 10000e^{10r} \]
\[ 2 = e^{10r} \]
\[ \ln 2 = 10r \]
\[ r = \frac{\ln 2}{10} \approx 0.0693 \]
\[ 6.93\% \]

2) How long will it take a bacteria culture of 200 mg to grow to 51,200 mg if it doubles every hour?

<table>
<thead>
<tr>
<th>n</th>
<th>y = bacteria amt (mg)</th>
<th>n = # hours</th>
<th>after 8 hrs.</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2(200) = 400 = 2^1(200)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2(400) = 800 = 2^2(200)</td>
<td></td>
<td>general</td>
</tr>
<tr>
<td>3</td>
<td>2(800) = 1600 = 2^3(200)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2(1600) = 3200 = 2^4(200)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2(3200) = 6400 = 2^5(200)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2(6400) = 12800</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2(12,800) = 25,600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2(25,600) = 51,200</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ y = 200(2^n) \]
\[ 51200 = 200(2^n) \]
\[ 256 = 2^n \]
\[ 256 = 2^8 \]
\[ n = 8 \]