Section 5.2: Adding and Subtracting Polynomials

Objectives:

- Identify leading coefficients and degrees of polynomials.
- Add and subtract polynomials using vertical and horizontal format.
- Use polynomials to model and solve real life problems.

\[ 5x^3 - 2x^2 + 3x + 6 \]
5.2b Adding and Subtracting Polynomials

Definition of a polynomial

\[ a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0 \]

Example

\[ 3x^4 - 5x^2 + 2x + 1 \]

Vocabulary

- **Degree**: \( n \)
  - Highest exponent (degree or power) on variable
- **Leading coefficient**: \( a_n \)
  - Coefficient of highest degree term
- **Constant term**: \( a_0 \)
  - The "plain" number

- **Binomial**: Two-term polynomial
- **Trinomial**: Three-term polynomial
- **Monomial**: One-term polynomial

Standard form

\[ \text{descending order} \]

State whether these are monomial, binomial or trinomial. State degree, leading coefficient and constant.

<table>
<thead>
<tr>
<th>Polynomials</th>
<th>Degree</th>
<th>Leading Coefficient</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( 3 - x^2 = -x^2 + 3 )</td>
<td>2</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>b) ( 4x^3 )</td>
<td>3</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>c) ( x^3 + 5x - 2 )</td>
<td>3</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

Are these polynomials? Why?

- a) \( x^{-2} + 7x - 2 \)
  - Not polynomial (dividing by \( x^2 \))
- b) \( \frac{1}{2x} - x + 1 \)
  - NOT polynomial
- c) \( \frac{2}{3}x^1 - 2x \)
  - IS polynomial (degree 1, leading coefficient \( \frac{2}{3} \), constant 0)

May 02, 2011
EXAMPLE

Combine like terms and put in standard form.

a) \((2x^4 + 3x^2 - x^2 + 5x + 7) + (3x^2 - x + 1)\)

\[
= 2x^4 + 3x^2 - x^2 + 5x + 7 + 3x^2 - x + 1 \\
= 2x^4 + 5x^2 + 4x + 8
\]

b) \((6t - 4t^3 - t^2 + 3) - (3t^2 - 50)\)

\[
= 6t - 4t^3 - t^2 + 3 - 3t^2 + 50 \\
= -4t^3 - 4t^2 + 6t + 53
\]
c) \((15 - 2y + y^2) + (3y^2 - 6y + 1) - (4y^2 - 8y + 16)\)

\[= 15 - 2y + y^2 + 3y^2 - 6y + 1 - 4y^2 + 8y - 16\]

\[= 0 \quad \text{degree: 0} \]

\[\phantom{=} \quad \text{constant: 0}\]

d) \((x^{2m} - 6x^m + 4) - (2x^{2m} - 4x^m - 3)\)

\[= x^{2m} - 6x^m + 4 - 2x^{2m} + 4x^m + 3\]

\[-x^{2m} - 2x^m + 7 \quad \text{degree: 2m} \quad \text{const = 7} \]

\[l.c. = -1\]
Application

Find an expression in terms of $x$ for the perimeter and for the area of this figure. Evaluate each if $x = 6$ ft.

If $x = 6$ ft,

$$P = 8(6) + 10 = 48 + 10 = 58 \text{ ft}.$$  
$$A = 18(6) = 108 \text{ ft}^2.$$