4.3: LINEAR SYSTEMS IN 3 VARIABLES

Objectives:

✧ Solve systems of equations in row-echelon form by back-substituting.
✧ Solve systems of equations using Gaussian elimination
✧ Solve application problems using Gaussian elimination.

\[
\begin{align*}
3x - 2y + 4z &= -8 \\
7y - 2z &= 6 \\
3z &= 12
\end{align*}
\]
Row echelon form for a system of equations:

\[ x - 2y + 3z = 9 \]
\[ y + 2z = 5 \]
\[ z = 3 \]

Three Elementary Row Operations:

1. Interchange two rows.
2. Multiply one row by a non-zero constant.
3. Add a multiple of one row to another row.

Use these operations to get this system of equations in row echelon form.
Possible solutions to a system of equations in three variables:

1. One answer
   - Solution: one point

2. Intersect at all points
   - Solution: one plane

3. Solution: none

4. Infinitely many intersection points
   - Solution: one line

Solution: none
EXAMPLE

Solve this system.

\[
\begin{align*}
  x - 2y + 2z &= 9 \\
  -x + 3y &= -4 \\
  2x - 5y + z &= 10
\end{align*}
\]
EXAMPLE:

Solve this system.

\[
\begin{align*}
-x + 3y - 2z &= -1 \\
x - 3y + z &= 1 \\
2x - y - 2z &= 2 \\
x + 2y - 3z &= -1
\end{align*}
\]
EXAMPLE:

Solve this system.

\[
\begin{align*}
    x + y - 3z &= -1 \\
    y - z &= 0 \\
    -x + 2y &= 1 \\
\end{align*}
\]

Graphically, these intersect in a line.

\[
\begin{align*}
    \text{(1)} & \quad x + y - 3z = -1 \\
    \text{(2)} & \quad y - z = 0 \\
\end{align*}
\]

\[
\begin{align*}
    \text{(1)} & \quad x + y - 3z = -1 \\
    \text{Multiply by } -3 \quad \Rightarrow & \quad -3y + 9z = 3 \\
    \text{(2)} & \quad y - z = 0 \\
    \text{Add } \quad \Rightarrow & \quad 3y - 3z = 0 \\
\end{align*}
\]

\[
\begin{align*}
    \text{Solve for } y & \quad \Rightarrow & \quad y = 3z \\
    \text{Substitute into } \text{(1)} & \quad \Rightarrow & \quad x + 3z - 3z = -1 \\
    \Rightarrow & \quad x = -1 \\
\end{align*}
\]

\[
\begin{align*}
    \text{Solve for } z & \quad \Rightarrow & \quad y = 3z \\
    \text{Substitute into } \text{(2)} & \quad \Rightarrow & \quad 3z - z = 0 \\
    \Rightarrow & \quad 2z = 0 \\
    \Rightarrow & \quad z = 0 \\
\end{align*}
\]

\[
\begin{align*}
    x = -1 \\
    y = 3z \\
    z = 0 \\
\end{align*}
\]

Line of intersection:

\[
\begin{align*}
    \begin{cases}
        x = 2z - 1 \\
        y = 3z \\
        z = z
    \end{cases}
\end{align*}
\]

\[
\begin{align*}
    (2z - 1, 3z, z)
\end{align*}
\]

\[
\begin{align*}
    x = 2a - 1 \\
    y = a \\
    z = a
\end{align*}
\]
EXAMPLE:
Write a set of equations to solve this problem.

The measure of one angle of a triangle is two-thirds the measure of a second angle. The measure of the second angle is $12^\circ$ greater than the measure of the third angle. Find the measures of the three angles of the triangle.

\[
\begin{align*}
\begin{cases}
 a = \frac{2}{3} b \\
 b = 12 + c \\
 a + b + c = 180
\end{cases}
\end{align*}
\]

Answer:
\[
\begin{align*}
 a &= 48^\circ \\
b &= 72^\circ \\
c &= 60^\circ
\end{align*}
\]