MATH 1010 ~ Intermediate Algebra

Chapter 4: SYSTEMS OF EQUATIONS

4.3: LINEAR SYSTEMS IN 3 VARIABLES

Objectives:

- ✧Solve systems of equations in row-echelon form by back-substituting.
- ✧Solve systems of equations using Gaussian elimination
- ✧Solve application problems using Gaussian elimination.

$$\begin{aligned} 3x - 2y + 4z &= -8 \\ 7y - 2z &= 6 \\ 3z &= 12 \end{aligned}$$

Row echelon form for a system of equations:

(triangular)

$$\begin{aligned} x - 2y + 3z &= 9 \\ y + 2z &= 5 \\ z &= 3 \end{aligned}$$

Three Elementary Row Operations:

1. Interchange two rows.
2. Multiply one row by a non-zero constant.
3. Add a multiple of one row to another row.

(elimination step)

Use these operations to get this system of equations in row echelon form.

(using elem. row ops)

$$\begin{aligned} x - 2y + 3z &= 5 \\ -x + y + 5z &= 4 \\ 2x \qquad - 3z &= 0 \end{aligned}$$

(-2) $-2x + 4y - 6z = -10$

$$\begin{aligned} x - 2y + 3z &= 5 \\ -y + 8z &= 9 \\ 2x \qquad -3z &= 0 \end{aligned}$$

$$\begin{aligned} x - 2y + 3z &= 5 \\ -y + 8z &= 9 \\ 4y - 9z &= -10 \end{aligned}$$

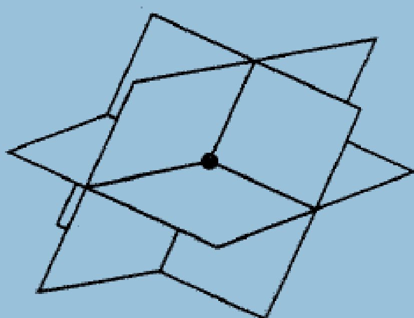
(4) $-4y + 32z = 36$

$$\begin{aligned} x - 2y + 3z &= 5 \\ -y + 8z &= 9 \\ 23z &= 26 \end{aligned}$$

now in row-echelon form

Possible solutions to a system of equations in three variables:

1 one answer



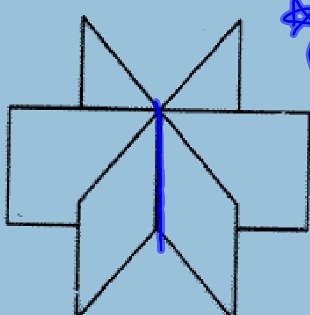
Solution: one point

2 intersect at all points

Solution: one plane

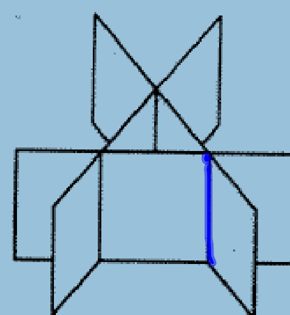
★ trickiest

4 infinitely many intersection pts

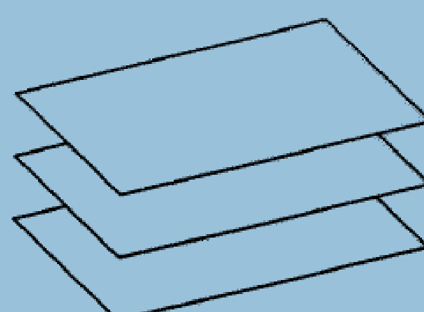


Solution: one line

3



Solution: none



Solution: none

① EXAMPLE

Solve this system.

$$x - 2y + 2z = 9$$
$$-x + 3y = -4$$
$$2x - 5y + z = 10$$

$-2x + 4y - 4z = -18$

$(-2)\ x - 2y + 2z = 9$

$y + 2z = 5$

$2x - 5y + z = 10$

$x - 2y + 2z = 9$

$y + 2z = 5$

$-y - 3z = -8$

① $x - 2y + 2z = 9$

② $y + 2z = 5$

③ $-z = -3$

③ $-z = -3$ $\longrightarrow$ ② $y + 2(3) = 5$

$z = 3$

$y + 6 = 5$

$y = -1$

① $x - 2(-1) + 2(3) = 9$

$x + 2 + 6 = 9$

$x + 8 = 9 \Rightarrow x = 1$

Soln: $(1, -1, 3)$

② EXAMPLE:

Solve this system.

$-2x + 6y - 2z = -2$

(-2) $x - 3y + z = 1$

$2x - y - 2z = 2$

$x + 2y - 3z = -1$

$-x + 3y - z = -1$

(-1) $x - 3y + z = 1$

$5y - 4z = 0$

$x + 2y - 3z = -1$

$x - 3y + z = 1$

$-5y + 4z = 0$

(-1) $5y - 4z = 0$

$5y - 4z = -2$

$x - 3y + z = 1$

$5y - 4z = 0$

$0 \neq -2$

$\Rightarrow$ N.S.

③ EXAMPLE:

Solve this system.

$$\begin{aligned} x + y - 3z &= -1 \\ y - z &= 0 \\ -x + 2y &= 1 \end{aligned}$$

$$x + y - 3z = -1$$

(-3) $$\begin{aligned} y - z &= 0 \\ -3y + 3z &= 0 \end{aligned}$$

$$3y - 3z = 0$$

① $x + y - 3z = -1$

② $y - z = 0$

$0 = 0$

★ graphically, these intersect in a line.

② $y - z = 0$

$y = z$

① $x + z - 3z = -1$

$x - 2z = -1$

$x = 2z - 1$

$z = z$

line of intersection: $\left\{ \begin{aligned} x &= 2z - 1 \\ y &= z \\ z &= z \end{aligned} \right\}$

$(2z-1, z, z)$

$x = 2a - 1$

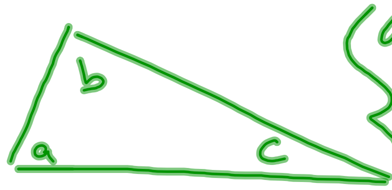
$y = a$

$z = a$

④ EXAMPLE:

Write a set of equations to solve this problem.

The measure of one angle of a triangle is two-thirds the measure of a second angle. The measure of the second angle is 12º greater than the measure of the third angle. Find the measures of the three angles of the triangle.



$$\begin{cases} a = \frac{2}{3}b \\ b = 12 + c \\ a+b+c = 180 \end{cases} \qquad \begin{cases} a - \frac{2}{3}b = 0 \\ b - c = 12 \\ a+b+c = 180 \end{cases}$$

Answer

$a = 48°$

$b = 72°$

$c = 60°$