Section 4.2: LINEAR SYSTEMS IN TWO VARIABLES

Objectives:

✧ Solve systems of equations by elimination.
✧ Use systems of equations to solve real life problems.

3 drinks + 4 doughnuts = $10.00
2 drinks + 2 doughnuts = $6.00

How much is 1 doughnut?
The method of elimination

1. Obtain coefficients for x (or y) that are opposites by multiplying all terms of one or both equations by suitable non-zero constants.

2. Add the equations to eliminate one variable and solve the resulting equation for the remaining variable.

3. Back-substitute the value obtained in step 2 in either of the original equations and solve for the other variable.

4. Check your solution in both of the original equations.

\[
\begin{align*}
1. & \quad 4x - 5y = 13 \\
2. & \quad 3x - 9y = 8 \\
3. & \quad 2x + 6y = 7
\end{align*}
\]

\[
\begin{align*}
\text{Step 1:} & \quad 4x - 5y = 13 \\
& \quad 3x - 9y = 8 \\
& \quad 2x + 6y = 7
\end{align*}
\]

\[
\begin{align*}
\text{Step 2:} & \quad 4x - 5y = 13 \\
& \quad -15x + 5y = -35
\end{align*}
\]

\[
\begin{align*}
-11x &= -22 \\
x &= 2
\end{align*}
\]

\[
\begin{align*}
\text{Step 3:} & \quad 4(2) - 5y = 13 \\
& \quad 8 - 5y = 13 \\
& \quad -5y = 5 \\
y &= -1
\end{align*}
\]

\[
\begin{align*}
\text{Solution:} & \quad (2, -1)
\end{align*}
\]

\[
\begin{align*}
\text{(choose to get rid of x)} & \quad 3x + 9y = 8 \\
& \quad 2x + 6y = 7
\end{align*}
\]

\[
\begin{align*}
\leftrightarrow & \quad 6x + 18y = 16 \\
& \quad + 2(-6x - 18y = -21)
\end{align*}
\]

\[
\begin{align*}
0 & \neq -5 \\
\Rightarrow & \quad N.S. \quad \text{(parallel lines)}
\end{align*}
\]
EXAMPLE:

Solve these systems by elimination.

1. a) \(-x + 2y = 9\)
   \(x + 3y = 16\)

   \[
   \begin{align*}
   &1 \quad 5y = 25 \\
   &2 \quad y = 5 \\
   \text{Soln: (} &1, 5\text{)}
   \end{align*}
   \]

2. b) \(3y = 2x + 21\)
   \(\frac{2}{3}x = 50 + y\)

   \[
   \begin{align*}
   &1 \quad -2x + 3y = 21 \\
   &2 \quad 3\left(\frac{2}{3}x - y\right) = 50 \quad \text{(Step 1)}
   \end{align*}
   \]

   \[
   \begin{align*}
   &1 \quad -2x + 3y = 21 \\
   &2 \quad 2x - 3y = 150 \quad \Rightarrow \text{N.S.} \quad \text{(parallel lines)}
   \end{align*}
   \]

3. c) \(4x = 6 + 5y\)
   \(8x = 12 + 10y\)

   \[
   \begin{align*}
   &1 \quad -8x = -12 + 10y \\
   &2 \quad 8x = 12 + 10y \quad \Rightarrow \text{same lines} \\
   &\quad \text{(every pt is the same)}
   \end{align*}
   \]
EXAMPLE:
Solve these applications by an appropriate method.

a) An SUV costs $26,445 and an average of $0.18 per mile to maintain. A hybrid model of the SUV costs $31,910 and $0.13 to maintain.

After how many miles will the cost of the SUV exceed the cost of the hybrid?

Cost of SUV: \[ 26445 + 0.18x = y \]
Cost of hybrid: \[ 31910 + 0.13x = y \]

\[ x = \text{when costs are equal} \]

\[ \frac{5435}{0.05} = 0 \]

\[ x = \frac{5435}{0.05} \]

b) A total of $1790 was made by selling 200 adult tickets and 316 children's tickets to a charity event. The next night a total of $937.50 was made by selling 100 adult tickets and 175 children's tickets.

Find the price of each type of ticket.

Price of child ticket: \( c \)
Price of adult ticket: \( a \)

\[ \begin{align*}
1790 &= 200a + 316c \\
2(937.5) &= 100a + 175c \\
1790 &= 200a + 316c \\
&+ -1875 = -200a - 350c \\
&- 85 = -34c \\
&\frac{-85}{-34} = c \\
&c = 2.50 \\
1790 &= 200a + 314(2.5) \\
1790 &= 200a + 790 \\
-790 &= -790 \\
1000 &= 200a \\
\boxed{a = 5}
\end{align*} \]