Objectives:

- Determine if ordered pairs are solutions of systems of equations.
- Solve systems of equations graphically.
- Solve systems of equations by substitution.
- Use systems of equations to model and solve real life problems.

\[ \begin{align*}
  x + y &= 3 \\
  x - y &= -2
\end{align*} \]
Vocabulary:

- **system of equations**: a collection of equations; typically we will have the same number of equations as variables.
- **solution**: for a system of equations, the solution is the point of intersection.
- **point of intersection**

Types of systems:

1. Independent and consistent
2. Parallel lines (inconsistent)
3. Same line (dependent)

Consistency:

- **consistent**: independent, same line (not the same line)
- **inconsistent**: parallel lines
- **dependent**: same line
Three methods to solve a system of equations:

1. Graphing: graph both curves on same axes (least used) and find the pt. of intersection

2. Substitution: use one of the eqns to solve for one variable; substitute that into the other eqn.

3. Elimination
EXAMPLE:
Solve each system by graphing

a) \begin{align*}
\text{1} & \quad x - y = 3 \\
\text{2} & \quad 2x + 3y = 7
\end{align*}

\begin{align*}
\text{1} & \quad x = y + 3 \\
& \quad y = x - 3 \\
\text{2} & \quad 3y = -2x + 7 \\
& \quad y = -\frac{2}{3}x + \frac{7}{3}
\end{align*}

b) \begin{align*}
\text{1} & \quad 2x + y = 3 \\
\text{2} & \quad 2y = -4x + 8
\end{align*}

\begin{align*}
\text{1} & \quad y = -2x + 3 \\
\text{2} & \quad y = -2x + 4 \\
& \text{N.S.}
\end{align*}
EXAMPLE

Solve by substitution

a) \[ \begin{align*}
y &= 2x + 1 \\
3x + 2y &= 16
\end{align*} \]

\[ \begin{align*}
\text{①} & \quad y = 2(2) + 1 \\
& \quad = 4 + 1 \\
& \quad = 5 \\
\text{soln: } (2, 5)
\end{align*} \]

b) \[ \begin{align*}
x + y &= 3 \\
2y &= 2x + 6
\end{align*} \]

\[ \begin{align*}
\text{②} & \quad y = \frac{x + 3}{2} \\
\Rightarrow & \quad 2y = x + 3 \\
\Rightarrow & \quad \text{①} \quad x + (x + 3) = 3 \\
\Rightarrow & \quad 2x + 3 = 3 \\
\Rightarrow & \quad \frac{2x}{2} = \frac{0}{2} \\
\Rightarrow & \quad x = 0 \\
\text{soln: } (0, 3)
\end{align*} \]

c) \[ \begin{align*}
2x + 5y &= 15 \\
y &= -\frac{2}{5}x
\end{align*} \]

\[ \begin{align*}
\text{①} & \quad 2x + 8\left(-\frac{2}{5}x\right) = 15 \\
& \quad 2x - \frac{16}{5}x = 15 \\
& \quad 0 \neq 15
\end{align*} \]

\[ \Rightarrow \text{N.S. (no soln)} \] (parallel lines)
a) \[ \begin{align*}
1) & \quad x - y = 5 \\
2) & \quad 2x = 2y + 10
\end{align*} \]

\[ \Rightarrow \text{same line} \]

b) \[ \begin{align*}
1) & \quad y = -\frac{3}{2}x + 4 \\
2) & \quad 3x + 2y = 3
\end{align*} \]

\[ \Rightarrow \text{N.S.} \]
EXAMPLE:
Set up a set of equations and solve these problems.

a) The sum of two numbers is 160.
The larger number is three times the smaller number.

\[ x = 1^{st} \text{ number}, \quad y = 2^{nd} \text{ number (larger)} \]
\[ x + y = 160 \quad \text{(1)} \]
\[ y = 3x \quad \text{(2)} \]
\[ 4x = 160 \Rightarrow x = 40 \]
\[ y = 3(40) = 120 \]

b) The perimeter of a rectangle is 90 meters.
The length is 1½ times the width.
Find the dimensions of the rectangle.

\[ P = 2b + 2h = 90 \quad \text{(1)} \]
\[ b = \frac{3}{2}h \quad \text{(2)} \]
\[ 2\left(\frac{3}{2}h\right) + 2h = 90 \]
\[ 3h + 2h = 90 \rightarrow 5h = 90 \Rightarrow h = 18 \]
\[ b = \frac{3}{2}(18) = 27 \]
c) Ten pounds of a nut mixture sells for $6.95 per pound. The mixture is made from two kinds of nuts; peanuts at $5.65 per pound and cashews at $8.95 per pound.

How many pounds of each will be used in the mixture?