

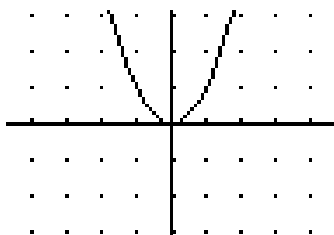
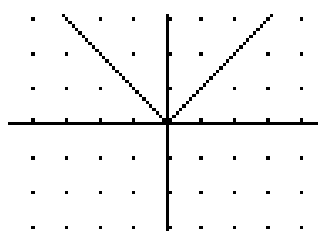
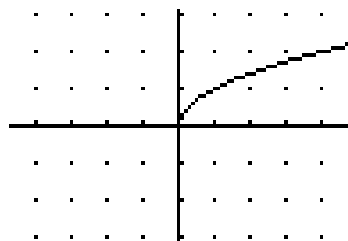
MATH 1010 ~ Intermediate Algebra

Chapter 3: Linear Equations and Inequalities

Section 3.7: Graphs of Functions

Objectives:

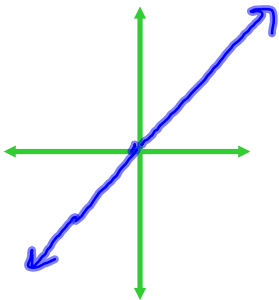
- * Sketch the graph of a function on a rectangular coordinate system.
- * Identify the graphs of basic functions.
- * Use the Vertical Line Test to determine if a graph is a function.
- * Use the vertical and horizontal shifts and reflections to sketch the graphs of functions.



We have already seen graphs of several basic functions.

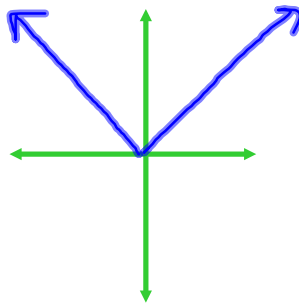
Linear

$$y = x$$



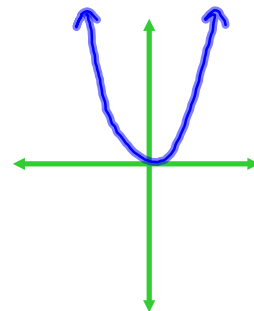
Absolute value

$$y = |x|$$



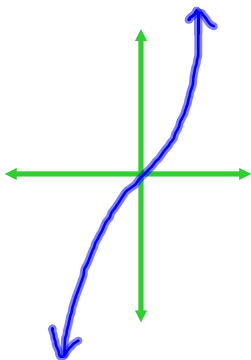
Parabola

$$y = x^2$$



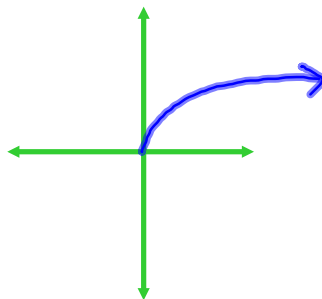
Cubic

$$y = x^3$$



Square root

$$y = \sqrt{x}$$

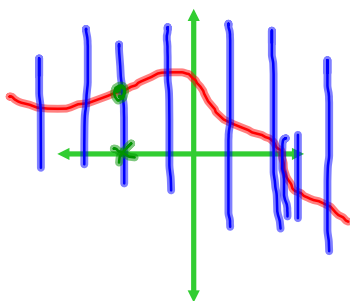


The Vertical line test states that a graph is a function if any vertical line only goes through at most one point on the graph.

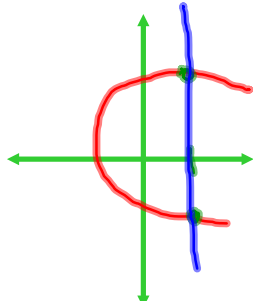
(because f_n has exactly one output for every input)

Examples: Function or not?

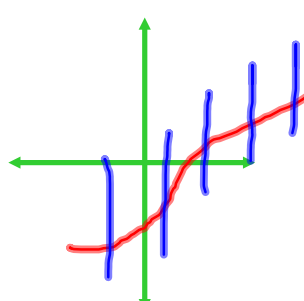
is f_n



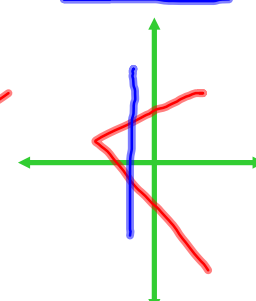
not f_n



is f_n



not f_n



Transformations of Graphs

vertical \Rightarrow outside the fn

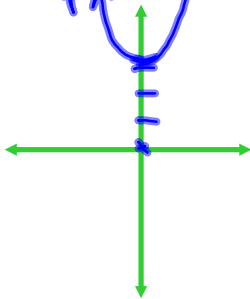
horizontal \Rightarrow inside the fn

Vertical shifts:

$$y = x^2 + 3$$

add/subtract a

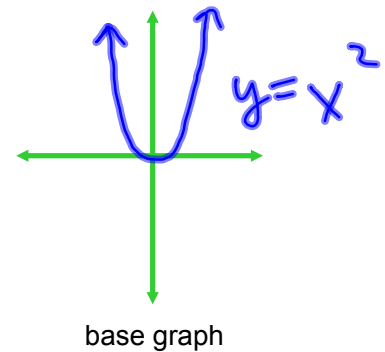
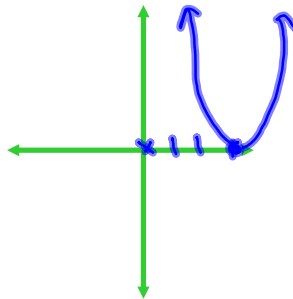
outside the fn
shift up 3



Horizontal shifts:

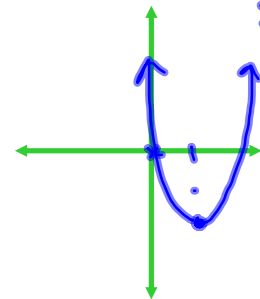
$$y = (x - 3)^2$$

add/subtract a #
inside the fn
shift right 3



$$y = (x - 1)^2 - 2$$

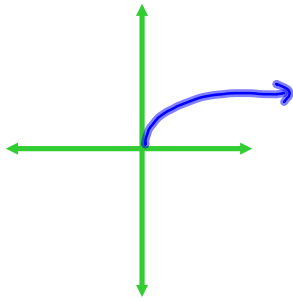
shifts
R 1
down 2



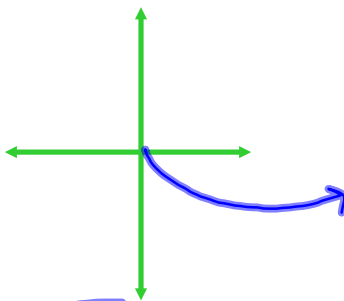
Reflection across the x-axis:

vertical reflection
multiplied by -1
outside

base $y = \sqrt{x}$



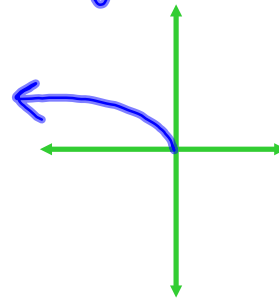
$y = -\sqrt{x}$



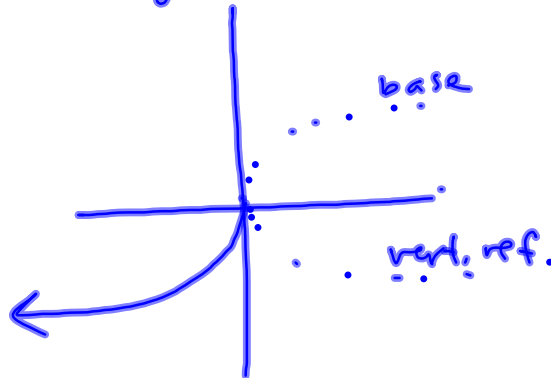
Reflection across the y-axis:

horizontal reflection
mult. by -1
inside

$y = \sqrt{-x}$



$y = -\sqrt{x}$

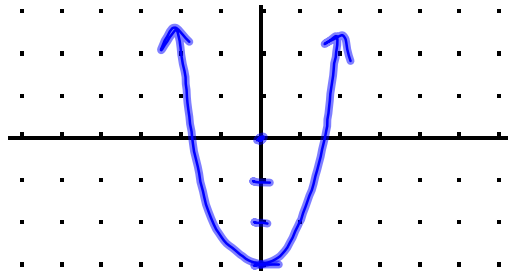


① EXAMPLE:

Sketch the graph. State the domain and range.

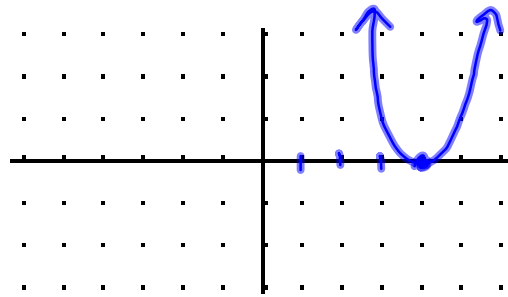
a) $f(x) = x^2 - 3$

base graph: $y = x^2$
 shift down 3
 domain: $x \in \mathbb{R}$
 range: $y \geq -3$



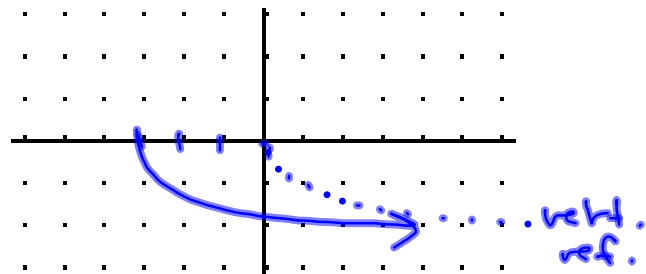
b) $f(x) = (x-4)^2$

base graph $y = x^2$
 horiz. shift right 4
 domain: $x \in \mathbb{R}$
 range: $y \geq 0$



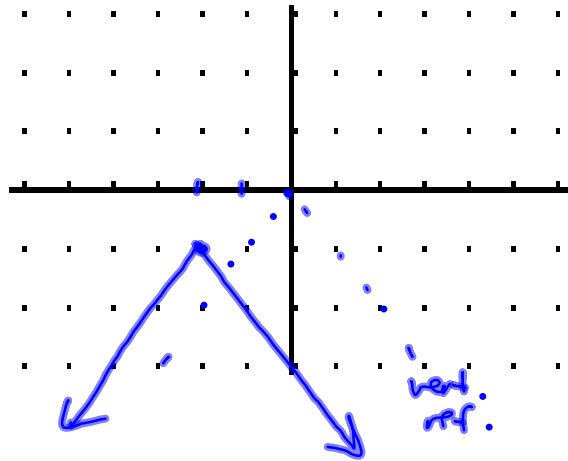
c) $f(x) = -\sqrt{x+3}$

base graph: $y = \sqrt{x}$
 horiz. shift left 3
 vert. reflection



domain: $x+3 \geq 0$
 $x \geq -3$
 range: $y \leq 0$

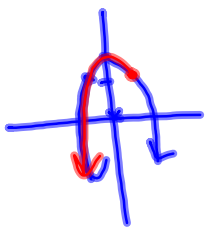
d) $g(x) = -|x+2|-1$
 base graph $y = |x|$
 vert. reflection
 horiz. shift left 2
 vert. shift down 1
 domain: $x \in \mathbb{R}$
 range: $y \leq -1$



e) $h(x) = \begin{cases} 2-x^2 & \text{if } x \leq 1 \\ x-2 & \text{if } x > 1 \end{cases}$

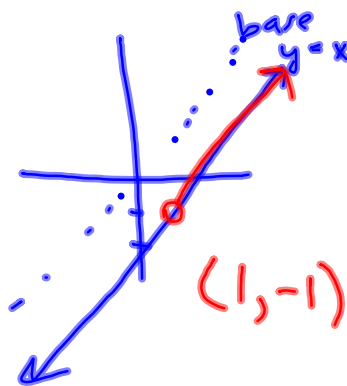
$h(x) = \begin{cases} \textcircled{1} -x^2 + 2 & x \leq 1 \\ \textcircled{2} x - 2 & x > 1 \end{cases}$

① $y = -x^2 + 2$
 base: $y = x^2$
 vert. reflection
 vert. shift up 2



$(0, 2)$

② $y = x - 2$



$(1, -1)$

domain:
 $x \in \mathbb{R}$

range: $y \in \mathbb{R}$
 $(-\infty, \infty)$