Section 3.4: Equations of Lines

Objectives:

✽ Write equations of lines using point-slope form.
✽ Write equations of horizontal, vertical, parallel and perpendicular lines.
✽ Graph a linear equation without changing the form of the equation.
✽ Use linear models to solve application problems.
Point-slope form of an equation of a line: \[ y-y_1 = m(x-x_1) \]

\((x_1,y_1)\) is a point on the line, \(m\) is the slope of the line.

Slope-intercept form of an equation is \[ y = mx + b \]

\(m\) is the slope and \((0,b)\) is the y-intercept.

General form of an equation of a line: \[ Ax + By + C = 0 \]

\(A, B,\) and \(C\) are integers.

Write the equation of a line with slope \(m = 3/5\) which goes through the point \((-1,2)\) and put it in each of the three forms.

\[
\begin{align*}
\text{Point-slope form:} & \quad y - 2 = \frac{3}{5}(x + 1) \\
& \quad \text{Simplifying:} \quad y - 2 = \frac{3}{5}x + \frac{3}{5} + 2 \\
& \quad \text{Solving for } y: \quad y = \frac{3}{5}x + \frac{13}{5} \\
& \quad \text{General Form:} \quad 5\left(\frac{3}{5}x + y - \frac{13}{5}\right) = 0 \\
& \quad \text{Multiplying through by 5:} \quad -3x + 5y - 13 = 0
\end{align*}
\]
EXAMPLE
Write the equation in slope-intercept form for the lines containing these pairs of points.

a) (-3,2) and (5,2)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{5 - (-3)} = \frac{0}{8} = 0 \]

\[ \text{Pt slope} \quad y - 2 = 0(x - (-3)) \]

(General Form)

\[ y - 2 = 0 \]

(Slope-intercept form)

\[ y = 2 \]

b) (-3,2) and (-3,5)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{-3 - (-3)} = \frac{3}{0} = \text{Undefined} \]

Vertical line \[ x = -3 \]

c) (-3/2, -1/2) and (5/8, 1/2)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{1}{2} - \frac{1}{2}}{\frac{5}{8} - \frac{-3}{2}} = \frac{0}{\frac{5}{8} + \frac{3}{2}(\frac{4}{4})} = \frac{1}{\frac{5}{8} + \frac{6}{8}} = \frac{1}{\frac{11}{8}} = \frac{8}{17} \]

Pt. slope form

\[ y - \left(-\frac{1}{2}\right) = \frac{8}{17}(x - \frac{3}{2}) \]

\[ y + \frac{1}{2} = \frac{8}{17}x + \frac{4}{17}(3) \]

\[ y + \frac{1}{2} = \frac{8}{17}x + \frac{12}{17} \]

\[ y = \frac{8}{17}x + \frac{12}{17} - \frac{1}{2} \]

\[ = \frac{12}{17} - \frac{1}{2} \]

\[ = \frac{12}{17} \left(\frac{1}{2}\right) - \frac{1}{2} \left(\frac{17}{17}\right) \]

\[ = \frac{12}{34} - \frac{17}{34} = \frac{7}{34} \]

Slope-intercept form

\[ y = \frac{8}{17}x + \frac{7}{34} \]
EXAMPLE

Write the equation of a line through (3,2) and (5,-4).
State the equation in point-slope form \((y-y_1) = m(x-x_1)\)
slope-intercept form \((y = mx+b)\) and
general form \((Ax + By + C = 0)\)

\[
m = \frac{-4-2}{5-3} = \frac{-6}{2} = -3
\]

\[
y-2 = -3(x-3)
\]
\[
y = -3x + 9
\]

\[
y = -3x + 11
\]

\[
3x + y = 11
\]
\[
3x + y - 11 = 0
\]
Horizontal and Vertical lines

A horizontal line has an equation in the form: $y = a$

A vertical line has an equation in the form: $x = b$

Example
Graph these equations and write the coordinates of three points on each line.

1. $x = -2$
2. $y = 3$
EXAMPLE

a) Write an equation of a vertical line through (5,8)

\[ x = 5 \]

b) Write an equation of a horizontal line through (-1, 7)

\[ y = 7 \]
EXAMPLE

Find the equation of a line perpendicular to \(3x - 4y = 12\) which passes through the point \((-3,6)\)

\[
m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{-4}{3} \quad \text{for the line through } (-3,6)
\]

\[
y - 6 = \frac{-4}{3} (x + 3)
\]

\[
y - 6 = \frac{-4}{3} x - 4
\]

\[
y = \frac{-4}{3} x + 2
\]
How to sketch a linear equation without changing the form of the equation.

\[ y = -2 \]

\[ 3x - 2y = 6 \]
\[ \Rightarrow y = -\frac{3}{2}x + 3 \]

\[ y = -\frac{2}{3}x \]

\[ x = 3 \]

\[ y - 3 = -2(x + 1) \]

\[ y = \frac{3}{2}x - 2 \]
Applications:

a) The total sales for a new sportswear store were $150,000 for the third year and $250,000 for the fifth year. Find a linear model to represent the data. Estimate the total sales for the sixth year.

\[ m = \frac{250000 - 150000}{5 - 3} = \frac{100000}{2} = 50000 \]

\[ y - 150000 = 50000(x - 3) \]

\[ y = 50000x - 150000 \]

b) A business purchases a van for $27,500. After 5 years the depreciated value will be $12,000. Assuming a straight-line depreciation, write an equation of the line giving the value \( V \) of the van in terms of the time \( t \) in years.

Use that equation to find the value of the van after 2 years.

\[ m = \frac{27500 - 12000}{0 - 5} = \frac{15500}{-5} = -3100 \]

\[ V - 27500 = -3100(t - 0) \]

\[ V = -3100t + 27500 \]

After 2 yrs, \( t = 2 \)

\[ V = -3100(2) + 27500 \]

\[ = -6200 + 27500 \]

\[ = 21300 \]