Section 2.3: Business and Scientific Problems

Objectives:

* Use mathematical models to solve business-related problems.
* Use mathematical models to solve mixture problems.
* Use mathematical models to solve rate problems.

It takes me 3 hours to perform a task; It takes my friend 5 hours. If we work together, how long should it take?
RATES IN BUSINESS

EXAMPLE:

Simple Interest:

(#86) Find the annual interest rate on a CD that earned $400 interest in 2 years on a principal of $2500.

\[ A = P(1+r) \quad \text{or} \quad I = Prt \]

\begin{align*}
A &= \text{amt after time} \\
P &= \text{principal (initial investment)} \\
r &= \text{interest rate (\%)} \\
I &= \text{interest ($)} \\
t &= \text{time (yrs)}
\end{align*}

\[ I = Prt \]

\[ 400 = 2500(r)(2) \]

\[ \frac{400}{5000} = \frac{r}{\cancel{5000}} \]

\[ \frac{4}{50} = r \]

\[ 8\% = 0.08 = \frac{8}{100} = r \]
(32) An appliance store charges $50 for the first 1/2 hour of a call and $18 for each additional 1/2 hour of labor. Find the length of service call if you were charged $104.

\[ x = \text{length of service call after 1st half hr} \]
\[ y = \text{length of service call (hours)} \]

\[ 104 = 50 + 18x \]
\[ 54 = 18x \]
\[ x = 3 \]
\[ 3(\frac{1}{2}) = \frac{3}{2} = 1.5 \text{ hrs} \]
\[ 1(\frac{1}{2} \text{ hr}) + 3(\frac{1}{2} \text{ hr}) = 4(\frac{1}{2} \text{ hr}) = 2 \text{ hrs total} \]
EXAMPLE:

A department store sells a beach towel for $14.00. On sale, the towel is $10.00. What is the discount rate?

\[
\frac{4}{14} \text{ is } \frac{x}{2} \text{ of } \frac{14}{14}
\]

\[
\frac{4}{14} = \frac{14x}{14} \quad \Rightarrow \quad x = 0.2857 \approx 28.572
\]

\[
\frac{2}{9} = x
\]
MIXTURE PROBLEMS

EXAMPLE:

A grocer mixes two kinds of nuts costing $3.88 per pound and $4.88 per pound to make 100 pounds of a mixture costing $4.13 per pound. How many pounds of each kind of nut are in the mixture?

\[
\begin{array}{c|c|c|c}
\text{nut 1} & \text{nut 2} & \text{mix} \\
\hline
\text{3.88}/\text{lb} & \text{4.88}/\text{lb} & \text{54.13}/\text{lb} \\
\hline
\text{\# lbs} & \text{100-x} & \text{100} \\
\hline
\text{\$} & \text{3.88x} & \text{4.88(100-x)} \\
\hline
\text{\$} & \frac{3.88x + 4.88(100-x)}{100} = 4.13 \\
\hline
\push{75 \text{ lbs nut 1}} & \push{25 \text{ lbs nut 2}}
\end{array}
\]

\[
3.88x + 4.88(100-x) = 413
\]

\[
-x + 4.88 = 413
\]

\[
x = 75
\]
EXAMPLE:

Ticket sales for a spaghetti dinner total $1350. There are 4 times as many adult tickets sold as children's tickets. The adult tickets are $6.00 and the children's are $3.00. Find the number of children's tickets sold.

\[
\begin{array}{ccc}
\text{adult} & \frac{6}{4}c & 4c & 6(4c) \\
\text{child} & \frac{3}{4}c & c & 3c \\
\text{total} & x & 5c & 1350 \\
\end{array}
\]

\[6(4c) + 3c = 1350\]
\[24c + 3c = 1350\]
\[27c = 1350\]
\[c = 50 \text{ tickets}\]
DISTANCE PROBLEMS

EXAMPLE:

You ride your bike at an average speed of 8 mi/hr. How long will it take you to ride 12 miles?

\[ d = r \cdot t \]
\[ \text{(mi) = \frac{\text{mi}}{\text{hr}}} \]
\[ t = \text{time} \]
\[ r = 8 \text{ mi/hr} \]
\[ d = 12 \text{ mi} \]

\[ 12 = 8t \]
\[ \frac{12}{8} = t \]
\[ 1.5 = \frac{3}{2} = t \]

1.5 hrs.
WORK-RATE PROBLEMS

EXAMPLE:

I can complete a typing task in 4 hours. My daughter can do the task in 7 hours. How long will it take us if we work together?

\[ t = \text{time it takes to complete job together} \]

\[ 28t \left( \frac{1}{4} + \frac{1}{7} \right) = \left( \frac{1}{t} \right) \cdot 28t \]

\[ \frac{\text{amt of job I do}}{1 \text{ hr}} + \frac{\text{amt of job daughter does}}{1 \text{ hr}} = \frac{\text{amt of job done together}}{1 \text{ hr}} \]

\[ \frac{7}{28t} + \frac{4}{28t} = \frac{28t}{t} \]

\[ 7t + 4t = 28 \]

\[ 11t = 28 \]

\[ t = \frac{28}{11} \text{ hr} = \frac{28}{11} \text{ hr} \approx 2.5 \text{ hr} \]
SOLVING FOR A VARIABLE IN A FORMULA

Solve for \( c \) in this formula. \( s = C + rC \)

\[
\frac{s}{1+r} = \frac{C(1+r)}{1+r} = C
\]