Section 1.3: Properties of Real Numbers

Objectives:

✱ Identify and use the properties of real numbers.
✱ Develop additional properties of real numbers.

Which properties are used?

\[ a(b+c) = ab + ac = ba + ca = ca + ba = (c+b)a \]
PROPERTIES OF REAL NUMBERS

Commutative Property:
1. **addition**: \( a + b = b + a \)  \( \text{ex} \ 2 + 3 = 3 + 2 \)
2. **multiplication**: \( ab = ba \)  \( \text{ex} \ 3(5) = 5(3) \)

Associative Property:
1. **addition**: \((a+b)+c = a+(b+c)\)
2. **multiplication**: \((ab)c = a(bc)\)

**Distributive Property of Multiplication over Addition/Subtraction**
\[ a(b+c) = ab + ac \]
\( \text{ex} \ 5(2+1) = 5(2) + 5(1) = 15 \)

Additive Identity Property:
\[ a + 0 = a \quad 0 = \text{additive identity} \]
\[ = 0 + a \]

Multiplicative Identity Property:
\[ a \cdot 1 = a = 1 \cdot a \quad 1 = \text{mult. identity} \]

Additive Inverse Property:
\[ a + (-a) = 0 \]
\( \text{ex} \ 5 + (-5) = 0 \)
\(-a\) is add. inverse of \(a\)

Multiplicative Inverse Property:
\[ a \left( \frac{1}{a} \right) = 1 \]
\( \text{ex} \ -3 \left( \frac{1}{3} \right) = 1 \)
\( \frac{1}{a} \) is mult. inverse of \(a\)
PROPERTIES OF EQUALITY (true for equations)

Addition Property of Equality: add same quantity to both sides of eqn.
3x - 1 = 4

Multiplication Property of Equality: multiply by same quantity on both sides of eqn (not zero)
3x - 1 + 1 = 4 + 1

Cancellation Property of Addition: add inverse to both sides of eqn.
5x + 2 = 0
+2 +2
5x = -2

Cancellation Property of Multiplication: multiply both sides of eqn by mult. inverse
\[
\frac{1}{5} \cdot 5x = -2 \cdot \frac{1}{5}
\]

x = \frac{-2}{5}
PROPERTIES OF ZERO

Multiplication Property of Zero:

\[ ab = 0 \text{, then } a = 0 \text{ or } b = 0. \]

Division Property of Zero:

\[ \frac{0}{a} = 0, \quad a \neq 0 \]

\[ \text{ex} \quad \frac{0}{3} = 0 \iff 0 = 3 \cdot 0 \]

Division by Zero is Undefined:

\[ \frac{5}{0} = ? \]
\[ \iff 5 = ? \cdot 0 \]
\[ \text{nothing works} \]
\[ \Rightarrow \frac{5}{0} \text{ undefined} \]

\[ \frac{0}{0} = ? \]
\[ \iff 0 = ? \cdot 0 \]
\[ \text{everything works} \]
\[ \Rightarrow \text{undefined} \]
PROPERTIES OF NEGATION

Multiplication by -1:

- \( -1 \cdot a = -a \)

Example:

- \( -1 \cdot -5 = 5 \)

Placement of Negative Signs:

- \( \frac{-2}{5} = \frac{2}{-5} = -\frac{2}{5} \)

Product of Two Opposites:

- \( -a \cdot -b = (-1 \cdot -1)ab = ab \)
EXAMPLE: Simplify and state the properties used.

a) \[10(2x) = (10 \cdot 2)x = 20x\] \text{Associativity of multiplication}

b) \[17 - 17 = 17 + (-17) = 0\] \text{additive inverse}

c) \[\left(-\frac{3}{4}\right)\left(-\frac{4}{3}\right) = \frac{\frac{3}{4} \cdot \frac{4}{3}}{1} = 1\] \text{mult. inverse}

d) \[6(x+3) = 6x + 18\] \text{Distributivity}

e) The additive inverse of -16 is \(-16\), the multiplicative inverse is \[\frac{1}{16}\].

f) The additive inverse of \(-\frac{2}{3}\) is \(\frac{2}{3}\), the multiplicative inverse is \[\frac{3}{2}\].

g) \[12(\$19.95) = 12(20 - 0.05) = 12(20) - 12(0.05) = 240 - 0.60 = \$239.40\] \text{Distributivity}

h) BEWARE \((a+b)^2 =
\[
(a+b)^2 \neq a^2 + b^2
\]
\[
(a+b)^2 = (a+b)(a+b)
= a(a+b) + b(a+b)
= a^2 + ab + ab + b^2
= a^2 + 2ab + b^2
\]