

2022 State Math Competition
Junior Exam
Version A

Instructions:

- Make sure to mark the version on your answer sheet.
- Correct answers are worth 5 points. Unanswered questions will be given 2 points. Incorrect answers will be worth 0 point. This means that it is not in your best interest to guess answers unless you have eliminated some possibilities.
- Fill in the answers on the answer sheet using a pencil or pen.
- Time limit: 75 minutes.
- When you are finished, please give the exam and any scrap paper to the test administrator.
- Good luck!

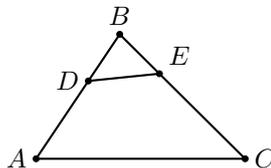
1. A company manufactures boxes that are 10 cm long, 8 cm wide, and 2 cm high. If the height of the box is increased by 50%, and the width is decreased by 25%, what is the percent increase in the volume of the box?

- (A) 5%
- (B) 10%
- (C) 20%
- (D) 37.5%
- (E) 12.5%

2. It is well known that Bojan is dangerous from within the paint as well as from behind the three-point arc. The number of 2-point shots he made last night was $\frac{5}{3}$ times as many as the number of 3-point shots he made. He also made 4 free throws (each worth 1 point). Knowing that he scored a total of 23 points, how many 3-point shots did he make?

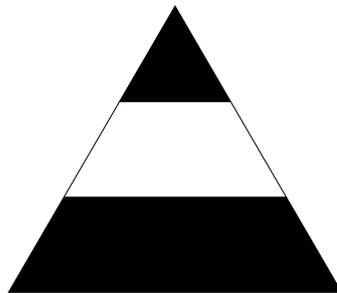
- (A) 4
- (B) 1
- (C) 5
- (D) 2
- (E) 3

3. In $\triangle ABC$, $\angle A = 35^\circ$ and $\angle C = 65^\circ$. D is on the side \overline{AB} , and E is on the side \overline{BC} so that $BD = BE$. What is the measure of $\angle BDE$?



- (A) 30°
- (B) 50°
- (C) 35°
- (D) 45°
- (E) 40°

4. For a natural number n , we define the factorial of n by $n! = n(n-1)(n-2)\cdots 1$. For example $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. The number $1! + 2! + 3! + \dots + 2022!$ has one's digit equal to
- (A) 3
 (B) 2
 (C) 8
 (D) 4
 (E) 9
5. The ratio $\frac{10^{2020} + 10^{2022}}{10^{2021} + 10^{2021}}$ is closest to which of the following numbers?
- (A) 5
 (B) 1
 (C) 10
 (D) 9
 (E) 0.5
6. Suppose we have two glasses of equal volume, glass A and glass B. Glass A is filled with milk while glass B is filled with water. One spoon of milk is transferred to glass B. Glass B is stirred, then one spoon of the mixture in glass B is transferred to glass A. Which of the following is true?
- (A) The volume of milk in glass B is smaller than the volume of water in glass A.
 (B) The information is insufficient to determine the answer.
 (C) The volume of milk in glass B is equal to the volume of water in glass A.
 (D) The volume of milk in glass B is bigger than the volume of water in glass A.
 (E) The volume of milk in glass A is smaller than the volume of water in glass A.
7. Consider an equilateral triangle with height 3. Cut it into three strips of equal height as shown in the figure below and color the top and bottom strip black. What is the total area of the black regions?



- (A) $2\sqrt{3}$
 (B) $\frac{\sqrt{3}}{2}$
 (C) $3\sqrt{2}$
 (D) $\frac{3}{\sqrt{2}}$
 (E) None of the above

8. Given that $i^2 = -1$, find $(1 + i)^{10} =$

- (A) 0
- (B) -32
- (C) $32i$
- (D) 32
- (E) $-32i$

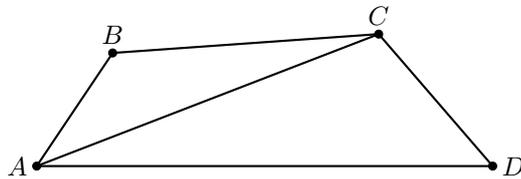
9. How many four-digit numbers of the form $abab$ are divisible by 16?

- (A) 1
- (B) 4
- (C) 3
- (D) 2
- (E) more than 4

10. Let a, b, c be real numbers such that $c(a + b + c) < 0$. Which of these is a possible value of $b^2 - 4ac$?

- (A) -37
- (B) 94
- (C) 0
- (D) -12
- (E) all of the above

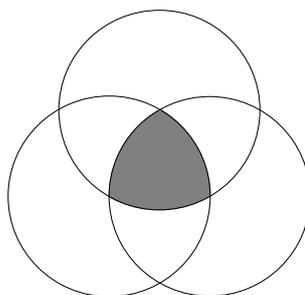
11. In quadrilateral $ABCD$, $AB = 6$, $BC = 10$, $CD = 7$, and $DA = 21$. If AC is an integer, then what is AC ?



- (A) 14
- (B) 15
- (C) 17
- (D) 16
- (E) 18

12. A sequence is generated using this procedure. The first number in the sequence is 1. To generate each successive term, a fair coin is flipped. If the result is heads, the next term is found by first multiplying the previous term by 2, and then subtracting 1. If the result is tails, the next term is found by first multiplying the previous term by 3, and then subtracting 1. What is the probability that the fourth term in the sequence is even?
- (A) $\frac{3}{8}$
(B) $\frac{2}{3}$
(C) $\frac{1}{4}$
(D) $\frac{1}{6}$
(E) $\frac{1}{2}$
13. In how many ways, we can rearrange the letters *BANANAS*?
- (A) 420
(B) 240
(C) 360
(D) 600
(E) 1260
14. The sides a, b, c of a triangle satisfy the relations $c^2 = 2ab$ and $a^2 + c^2 = 3b^2$. Then, $\angle BAC =$
- (A) 45
(B) 90
(C) 60
(D) 30
(E) impossible to determine
15. A new sequence is obtained from the sequence of the positive integers by deleting all the perfect squares. The 2022nd term of the new sequence is
- (A) 2067
(B) 2064
(C) 2066
(D) 2068
(E) 2065
16. What is the coefficient of x^3 in the product $(x + 1)(x + 2)(x + 2)(x + 3)(x + 3)$?
- (A) 47
(B) 39
(C) 97
(D) 38
(E) 27

17. An equilateral triangle has side length 2. What is the area of the region that contains all points outside the triangle but not more than 1 unit away from a point in the triangle?
- (A) $6 + \sqrt{3}\pi$
 (B) $3 + 3\sqrt{2}$
 (C) $6 + \pi$
 (D) $6 + \sqrt{3}$
 (E) $3 + 3\pi$
18. Seven points are selected on a circle. Three of the chords joining pairs of points are selected at random. What is the probability that the three chords form a triangle?
- (A) $\frac{1}{38}$
 (B) $\frac{1}{21}$
 (C) $\frac{3}{49}$
 (D) $\frac{2}{9}$
 (E) $\frac{1}{19}$
19. $1^2 - 2^2 + 3^2 - 4^2 + \dots + (101)^2 - (102)^2$ is equal to
- (A) -4851
 (B) -5253
 (C) -5050
 (D) -4685
 (E) -5464
20. Consider three circles of radius 1 which pass through each other's centers as shown in the figure below. What is the area of the shaded region enclosed by all of them?



- (A) $\frac{\pi}{4} - \frac{\sqrt{3}}{4}$
 (B) $\frac{\sqrt{3}}{4}$
 (C) $\frac{\pi}{2} - \frac{\sqrt{3}}{2}$
 (D) $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$
 (E) None of the above