Name: ____________________

School: ____________________

Grade: ____________________

2020 State Math Contest (Senior Exam)

Weber State University

March 4, 2020

Instructions:

- Do not turn this page until your proctor tells you.
- Enter your name, grade, and school information following the instructions given by your proctor.
- Calculators are not allowed on this exam.
- This is a multiple choice test with 40 questions. Each question is followed by answers marked (a), (b), (c), (d), and (e). Only one answer is correct.
- Mark your answer to each problem on the bubble sheet Answer Form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.
- Scoring: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
- You will have 2 hours and 30 minutes to finish the test.
- You may not leave the room until at least 10:30 a.m.

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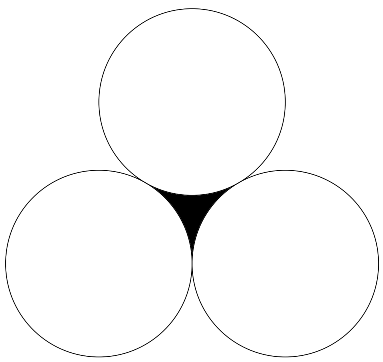
1. List all the possible values of $\frac{x}{|x|} + \frac{y+1}{|y+1|} + \frac{xz}{|xz|} + \frac{x+y+z}{|x+y+z|}$, where $x, y \in \mathbb{R}$.

   (a) $\{-4, 4\}$
   (b) $\{-4, 0, 4\}$
   (c) $\{-4, -2, 0, 2, 4\}$
   (d) All real numbers
   (e) None of the above

2. The radius of each of the three circles shown is r. What is the shaded area (between the circles)?



   (a) $\frac{\sqrt{3}}{2}\pi r^2$
   (b) $(\sqrt{2} - \frac{\pi}{4})r^2$
   (c) $\frac{\pi r^2}{2}$
   (d) $(\sqrt{5} - \frac{\pi}{3})r^2$
   (e) $(\sqrt{3} - \frac{\pi}{2})r^2$

3. There are five people in a room. What is the probability at least two individuals are born in the same month?

   (a) 100%
   (b) 61.8%
   (c) 38.19%
   (d) 38%
   (e) 0%

4. If $(1,4) \in A \cap B$ and $A = \{(x,y)|a^2x - b^2y - a = 0\}$ with $B = \{(x,y)|ax + b\sqrt{y} = 1\}$, then $A \cap B =$

   (a) $A \cap B = \{(x,y)|x = 1, y = 4\}$
   (b) $A \cap B = \{(x,y)|x = 1, y = 1\}$
   (c) $A \cap B = \{(x,y)|x = 1\}$
   (d) $A \cap B = \{(x,y)|y = 4\}$
   (e) None of the above

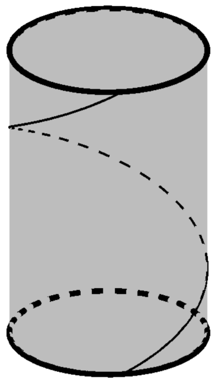
5. The sides of a triangle are p, $p+1$, and $2p-1$. Its area is $2p\sqrt{10}$. Find p.

   (a) 10
   (b) 11
   (c) 21
   (d) 31
   (e) None of the above

6. If $f(x) = \frac{e^x-1}{e^x+1}$, then

   (a) f is odd and the range of f is $(-1,1)$
   (b) f is odd and the range of f is $(-1,\infty)$
   (c) f is even and the range of f is $(-1,\infty)$
   (d) f is even and the range of f is $(-1,1)$
   (e) None of the above

7. A can of soup has a radius of 1 and a height of 2π. As a bug makes its way from the bottom of the can to the top, it follows the path of a helix, making one complete revolution and ending in a spot directly above where it began. (See diagram below.) How far does the bug travel?



(a) 4π

(b) $\frac{\sqrt{3}}{2}\pi$

(c) $4\pi^2$

(d) $2\sqrt{2}\pi$

(e) $2\sqrt{3}\pi$

8. What time does a 12-hour clock read 50 hours before it reads 6 o'clock?

(a) 4 o'clock

(b) 5 o'clock

(c) 2 o'clock

(d) 11 o'clock

(e) None of the above.

9. Each box of cereal contains a toy. The toy comes in 4 difference colors: red, orange, green or blue. Each of the 4 colors is equally likely to occur. Alex wants to get a toy in each of the 4 colors. If Alex buys 5 boxes of cereal, what is the probability that they get at least one toy of each of the 4 colors?

(a) $\frac{5}{1024}$

(b) $\frac{1}{120}$

(c) $\frac{1}{64}$

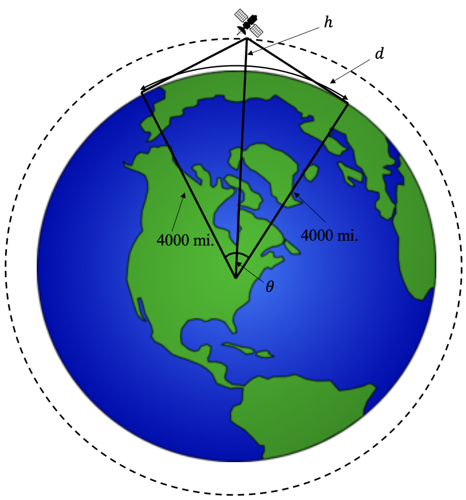
(d) $\frac{15}{64}$

(e) $\frac{1}{5}$

10. Suppose that $f(x) = x(x+1)(x+2)(x+3)\cdots(x+50)$. What is the coefficient of x^{50}?

    (a) 1275
    (b) 1350
    (c) 1375
    (d) 1400
    (e) 1425

11. A surveillance satellite circles Earth at a height of h miles above the surface. Suppose that d is the distance, in miles, on the surface of Earth that can be observed from the satellite. See the following diagram below, which is not to scale.



    If the satellite is tasked to observe a distance (d) of 4,186.67 miles, approximately how high is the satellite above Earth's surface (h)?

    (a) 50 miles
    (b) 100 miles
    (c) 300 miles
    (d) 500 miles
    (e) 700 miles

12. A company has a storage tank for vehicle fuel. The tank is a right circular cylinder with its axis parallel to the ground. The radius is 2 feet and the length is 8 feet. If the tank is full on day 1 and the depth on day 20 is 1 foot, then determine the average daily consumption to the nearest tenth of a ft^3/day.

    (a) 3.8 ft^3/day
    (b) 4.0 ft^3/day
    (c) 4.5 ft^3/day
    (d) 4.8 ft^3/day
    (e) 5.1 ft^3/day

13. A 1-meter measuring stick is cut at a randomly selected location, creating two sticks. What is the probability that the larger of the two sticks is over three times as long as the shorter of the two sticks, given that the longer of the two sticks is over twice as long as the shorter of the two sticks?

    (a) $\frac{1}{4}$
    (b) $\frac{1}{3}$
    (c) $\frac{1}{2}$
    (d) $\frac{2}{3}$
    (e) $\frac{3}{4}$

14. Suppose that $(x, y) = (5, 8)$ and $(x, y) = (3, 4)$ are solutions to the linear system of equations $\left\{ \begin{array}{ccc} ax + by & = & e \\ cx + dy & = & f \end{array} \right\}$, where a, b, c, d, e, and f are real numbers. Which of the following is also a solution to this same system of equations?

    (a) $(6, 6)$
    (b) $(2, 8)$
    (c) $(4, 7)$
    (d) $(7, 12)$
    (e) $(6, 9)$

15. Simplify the following: log(log(log(googolplex)))

    (a) 2
    (b) 3
    (c) 10
    (d) 1 million
    (e) A googol

16. An acute angle is formed by two lines of slope 1 and 7. What is the slope of the line which bisects this angle?

    (a) 2
    (b) 3
    (c) 1
    (d) 4
    (e) None of the above

17. Washington School sent their 4 best chess players to challenge the 4 best chess players from Lincoln School. Each of the 4 Washington players will be randomly paired with a different Lincoln player for the first round of games. How many different possible pairings are possible for the first round of games?

    (a) 4
    (b) 16
    (c) 24
    (d) 576
    (e) 40,320

18. Find the exact value of $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}$.

    (a) 1
    (b) 2
    (c) -1
    (d) 3
    (e) 6

19. An octahedron is formed by connecting the centers of the faces of a cube. What is the ratio of the volume of the cube to that of the contained octahedron?

    (a) $4:1$
    (b) $6:1$
    (c) $3:2$
    (d) $5:1$
    (e) None of the above

20. The police were informed of a car that was speeding. The license plate had six entries that could be either numbers or letters. The person reporting the car said that the first three entries of the license plate were digits and the last three were letters. The person saw that the first digit was 8. The last three letters comprised of "WHY", but the person could not remember the order of the letters. How many license plates must the police check?

    (a) 100
    (b) 600
    (c) 6
    (d) 60
    (e) 300

21. A cardboard box has a square base, with each edge of the base having a length of x inches. The total length of all 12 edges is 16 inches. How long should x be to maximize the volume?

    (a) $\frac{1}{3}$ inches
    (b) $\frac{2}{3}$ inches
    (c) 1 inch
    (d) $\frac{4}{3}$ inches
    (e) None of the above

22. Convert the base 7 number 2641.53 to base 10. What is the digit in the tenths position?

    (a) 3
    (b) 4
    (c) 6
    (d) 7
    (e) 8

23. A family just moved into a new house with an octagon pool. It is 10 feet deep and 20 feet across (from vertex to opposite vertex). The family wants to fill it up, but needs to know how much water it will take. What is the volume of the pool?

    (a) $1000\sqrt{2}$ ft^3
    (b) $2000\sqrt{2}$ ft^3
    (c) $4000\sqrt{2}$ ft^3
    (d) $8000\sqrt{2}$ ft^3

24. A student has a row of books that she is interested in reading. There are 3 books on skiing, 5 on mathematics, and 4 books on statistics. She would like to keep these books together on a single row on her book shelf. The mathematics and statistics books should be kept back to back, but we may permute the individual books within topics. How many different permutations of the books are possible?

    (a) 103,680
    (b) 17,280
    (c) 69,120
    (d) 34,560
    (e) 17,280

25. In triangle ABC, $AC = 6$ and $BC = 5$. Point D on AB divides it into segments of length $AD = 1$ and $DB = 3$. What is the length CD?

   (a) 11
   (b) $\frac{15}{2}$
   (c) $\frac{11}{2}$
   (d) 15
   (e) None of the above

26. Suppose $y'' + y = 0$. Which of the following is a possibility for $y = f(x)$?

   (a) $y = \tan x$
   (b) $y = \sec x$
   (c) $y = e^x$
   (d) $y = \sin x$
   (e) $y = \frac{1}{x}$

27. What is the number of real-valued solutions of the equation $x^2 + 21 = 10|x|$.

   (a) 4
   (b) 2
   (c) 0
   (d) 6
   (e) None of the above

28. Let n be greatest integer that will divide the three integers 15482, 15678, and 16168 and leave the same remainder. Which is the sum of the digits of n?

(a) 5
(b) 17
(c) 18
(d) 21
(e) 25

29. A person's blood pressure, P, varies with the cycle of their heartbeat. The pressure (in mmHg) at time t seconds for a particular person may be modeled by the function: $P(t) = 100 + 20\cos(2\pi t)$ mmHg, $t \geq 0$. According to this model, which of the following statements is accurate?

(a) The maximum pressure is 100 mmHg.
(b) The pressure goes through one complete cycle in 2 seconds.
(c) The amplitude of the pressure function is 120 mmHg.
(d) The pressure will reach a maximum value at time $t = 1$ second.
(e) Both statements (b) and (d) are accurate.

30. Let f be a continuous function on $[a, b]$ and differentiable on (a, b) such that $f(b) = 10$ and $f(a) = 2$. On which of the following intervals $[a, b]$ would the Mean Value Theorem guarantee a $c \in (a, b)$ such that $f'(c) = 4$?

(a) $[0, 4]$
(b) $[0, 3]$
(c) $[2, 4]$
(d) $[1, 10]$
(e) $(0, \infty)$

31. Two roots of the polynomial $3x^3 + \alpha x^2 - 5x - 10$ are r and $-r$ for some real number r. What is the value of α?

(a) 1
(b) 2
(c) -1
(d) 3
(e) 6

32. A ship has spotted an island and is approaching its midpoint. It's distance from the island is about 9,130 meters and the angle it subtends is 120°. What is the estimated width of the island?

(a) 4,565 meters
(b) 9,130 meters
(c) 18,260 meters
(d) $9{,}130\sqrt{3}$ meters
(e) $18{,}260\sqrt{3}$ meters

33. Find the length of the curve $x = 3t^2$, $y = 2t^3$, $0 \leq t \leq 1$.

(a) $2\sqrt{2} - 2$
(b) $4\sqrt{2} - 2$
(c) $4\sqrt{2}$
(d) $4\sqrt{2} - 1$
(e) None of the above.

34. Find exact value of the product $\log_c a \cdot \log_a b \cdot \log_b c$, if $\log_c b = 3$.

(a) 3
(b) 27
(c) 9
(d) 1
(e) None the above.

35. What is the exact value of $2(\sec\theta - \tan\theta)^2$, if $\frac{1-\sin\theta}{1+\sin\theta} = 3$?

(a) 3
(b) 9
(c) 1
(d) 5
(e) None of the above

36. Find the sum of the following infinite series: $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots$.

(a) 0.5
(b) 1
(c) 2
(d) 0.968
(e) 0.937

37. What is the greatest whole number that is a factor of the sum of any four consecutive positive even integers?

(a) 1
(b) 2
(c) 4
(d) 8
(e) 16

38. Suppose that $2x + \sqrt{2x + \sqrt{2x + \sqrt{2x + \cdots}}} = 2$, then x is

(a) $2 + \sqrt{2}$
(b) $2 - \sqrt{2}$
(c) $2 - \sqrt{3}$
(d) $2 + \sqrt{3}$
(e) None of the above.

39. The two shortest sides of a right triangle have lengths 3 and $\sqrt{7}$. Let α be the smallest angle of the triangle. What is $\cos \alpha$?

(a) $\frac{3}{4}$
(b) $\frac{\sqrt{7}}{3}$
(c) $\frac{\sqrt{7}}{4}$
(d) $\frac{2}{3}$

40. How many pairs of positive integers (x, y) satisfy the equation $2x + 11y = 340$?

(a) 15
(b) 10
(c) 20
(d) 5