Name: ______________________________

Student ID: ______________________________

State Math Contest (Junior)

Instructions:

- Do not turn this page until your proctor tells you.
- Enter your name, grade, and school information following the instructions given by your proctor.
- Calculators are not allowed on this exam.
- This is a multiple choice test with 40 questions. Each question is followed by answers marked a), b), c), d), and e). Only one answer is correct.
- Mark your answer to each problem on the bubble sheet Answer Form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.
- Scoring: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
- You will have 2 hours and 30 minutes to finish the test.
- You may not leave the room until at least 10:15 a.m.

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1. Zoey read 15 books, one at a time. The first book took her one day to read, the second book took her 2 days to read, the third book took her 3 days to read, and so on, with each book taking her 1 more day to read than the previous book. Zoey finished the first book on a Monday, and the second on a Wednesday. On what day the week did she finish her 15th book?

   a) Sunday b) Monday c) Wednesday

   d) Friday e) Saturday

   Correct answer: b) Monday.
   Solution: The process took $1 + 2 + 3 + 4 + 5 + \ldots + 15 = 120$ days, so the last day was 119 days after the first day. Since 119 is divisible by 7, both must have been the same day of the week, so the answer is b) Monday.

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2. There are four novels that you want to read over the summer. Two of the novels are by the same author and you don't want to read those directly after one another. In how many orders can you read the four novels?

   a) 4 b) 6 c) 12

   d) 18 e) 24

   Correct answer: c) 12.
   Solution: There are $4! = 24$ orders in which to read the four novels if there were no restrictions. The orders that are not acceptable are when the two novels by the same author are read directly after one another. The number of orders that are not acceptable can be calculated by letting the two novels by the same author be one item, then you arrange 3 items (the combined pair of novels by the same author and the other 2 novels) in 3! orders, and then there are 2 orders to read the two novels by the same author, for a total of $3! \times 2 = 12$ orders that are not acceptable. There are $24 - 12 = 12$ orders that are acceptable.

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3. Hans Larsen flew his plane 400 km with the wind in the same time it took him to fly 200 km against the wind. The speed of the wind is 50 km per hour. Find the speed of the plane in still air.

   a) 120 km/hr b) 150 km/hr c) 165 km/hr

d) 180 km/hr   e) 200 km/hr

Correct answer: b) 150 km/hr.
Solution: Since it took the same time to fly 400 km with the wind as it did to fly 200 km against the wind, the ratios of distance and rate of speed should be set equal to each other.

$$\frac{d_{with\ wind}}{(r_{plane}+r_{wind})_{with\ wind}} = \frac{d_{against\ wind}}{(r_{plane}-r_{wind})_{with\ wind}}$$
$$\frac{400}{(plane+50)} = \frac{200}{(plane-50)}$$

Solving the equation for the rate of speed of the plane produces the answer of 150 km per hour for the speed of the plane in still air.

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4. If two fair dice are rolled, what is the probability that the sum of the two dice is at least 4?

a) $\frac{1}{12}$ b) $\frac{5}{36}$ c) $\frac{11}{12}$

d) $\frac{1}{6}$ e) $\frac{4}{12}$

Correct answer: c) $\frac{11}{12}$.
Solution: The sum needs to be at least 4 and so we want sums of 4, 5, 6, 7, 8, 9, 10, 11, 12. There are 36 possible combinations for sums (6 total on each dice). Think of the number of ways to get 2 and 3, then subtract that from 1. There is one way to get 2 and two ways to get 3. That is three total ways out of 36. $\frac{3}{36} = \frac{1}{12}$ and $1 - \frac{1}{12} = \frac{11}{12}$.

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5. Given the point A = (–2, 6) and the line $y = 2x$, what are the coordinates of the point B obtained by reflecting A over the line $y = 2x$?

a) B = (7, 2)   b) B = (6, 3)   c) B = (5, 2.5)

d) B = (7, 1.5)   e) B = (6, 2)

Correct answer: e) B = (6, 2).
Solution: Note that the midpoint of the segment joining (–2, 6) and (6, 2) is the point (2, 4), which lies on the line $y = 2x$. Moreover, the slope of the line joining (–2, 6) and (6; 2) is $-\frac{1}{2}$, which is the negative reciprocal of 2. Thus the line $y = 2x$ is the perpendicular bisector of the line joining (–2, 6) and (6, 2).

6. The equation for the directrix of a certain parabola in the *xy*-plane is given by $y = -1$. If the vertex of the parabola is at (7, 3) what is the equation of the parabola?

a) $16(y-7)=(x-3)^2$ b) $16(y-3)=(x-7)^2$ c) $12(y-4)=(x-7)^2$

d) $14(y-3)=(x-4)^2$ e) $4(y-3)=(x-4)^2$

Correct answer: b) $16(y-3)=(x-7)^2$.
Solution: The distance *p* between the directrix and the vertex is 4. We use the vertex form of the equation for a parabola $4p(y-k)=(x-h)^2$ with $p=4$, $h=7$, and $k=3$.

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7. A cold-water faucet can fill a sink in 12 minutes, and a hot-water faucet can fill the same sink in 15 minutes. The drain at the bottom of the sink can empty the sink in 25 minutes. If both faucets and the drain are open, how long will it take to fill the sink?

a) $5\frac{15}{57}$ minutes b) $7\frac{3}{4}$ minutes c) $9\frac{1}{11}$ minutes

d) $10\frac{4}{11}$ minutes e) $20\frac{4}{57}$ minutes

Correct answer: c) $9\frac{1}{11}$ minutes.
Solution: If both faucets are working together to fill the sink, their work rates are added together. However, the drain is working against the faucets so that work rate needs to be subtracted from the faucets work rate.

$$\frac{time}{cold\text{-}water\ rate}+\frac{time}{hot\text{-}water\ rate}-\frac{time}{drain\ rate}=1$$
$$\frac{time}{12}+\frac{time}{15}-\frac{time}{25}=1$$

Solve the rational equation for the time needed to fill the sink by clearing out the denominators (LCD) and solving the equation. The resulting time to fill the sink is $9\frac{1}{11}$ minutes.

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8. How many ways can you write 7 as the sum of one or more positive integers if different orders are not counted differently? For example, there are three ways to write 3 in this way: 3, 2 + 1, and 1 + 1 + 1.

a) 6 b) 7 c) 13

d) 14 e) 15

Correct answer: e) 15.
Solution:

1) 7
2) 6 + 1
3) 5 + 2
4) 5 + 1 + 1
5) 4 + 3
6) 4 + 2 + 1
7) 4 + 1 + 1 + 1
8) 3 + 3 + 1
9) 3 + 2 + 2
10) 3 + 2 + 1 + 1
11) 3 + 1 + 1 + 1 + 1
12) 2 + 2 + 2 + 1
13) 2 + 2 + 1 + 1 + 1
14) 2 + 1 + 1 + 1 + 1 + 1
15) 1 + 1 + 1 + 1 + 1 + 1 + 1

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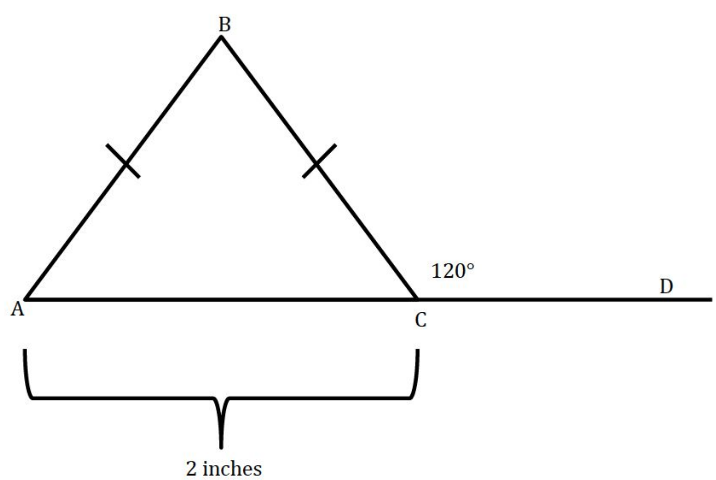
9. What is the area of triangle ABC shown below where $AB = BC$, $AC = 2$ inches, and $\angle BCD = 120°$?



a) $\sqrt{3}$ square inches b) $2\sqrt{3}$ square inches c) 1 square inch

d) 2 square inches e) 4 square inches

Correct answer: a) $\sqrt{3}$ square inches.
Solution: $\angle BCA = 60°$ and since $AB = BC$, $\angle BAC = 60°$, leaving $\angle ABC = 60°$. Thus the triangle is an equilateral triangle and all of the side lengths are 2 inches. Using the Pythagorean Theorem, solve for the height of the triangle:
$2^2 = 1^2 + H^2 \rightarrow 4 - 1 = H^2 \rightarrow H = \sqrt{3}$ inches. To find the area, find
$\frac{1}{2}BH = \frac{1}{2} \cdot 2 \cdot \sqrt{3} = \sqrt{3}$ square inches.

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10. Ryan, Eric, and Kim like to hike mountains. Kim hiked six mountains that neither Ryan nor Eric hiked. There were only four mountains that all three hikers climbed. There was only one mountain Eric and Ryan hiked that Kim did not hike. There were no mountains that only Eric and Kim hiked. Ryan hiked six times more mountains than Eric. Kim hiked $\frac{1}{3}$ of the mountains that Ryan hiked and two times more mountains that Eric hiked. How many mountains did only Ryan hike?

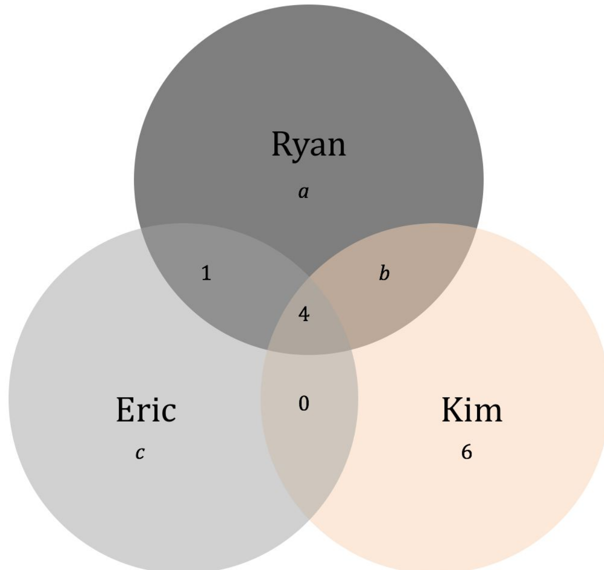
    a) 22   b) 27   c) 29

    d) 48   e) 56

    Correct answer: c) 29.

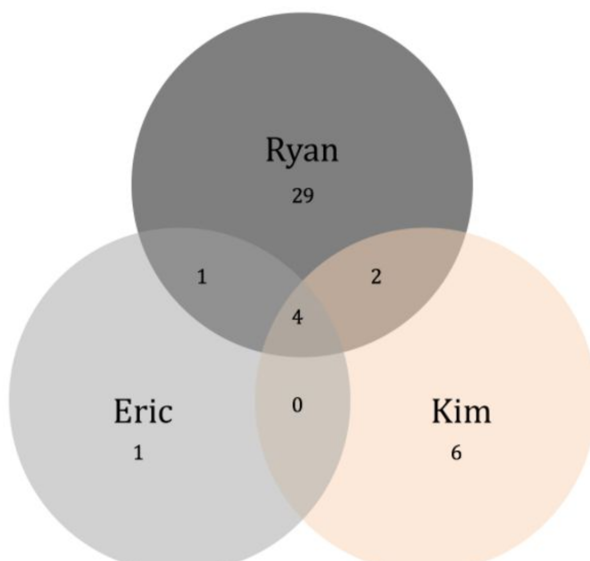
    Solution: From the problem, we know that Ryan hiked the most mountains, then Kim, then Eric. Ryan hikes six times as much as Eric, three times as much as Kim, and Kim hikes twice as much as Eric. So, if we find how many mountains Eric hikes, we can multiply that number by two for Kim's amount and by six for Ryan's amount.

    Using a Venn diagram, we know the following information and we use variables for the information we don't know:



If one were to use a system of equations to solve this problem, they would all be consistent and not helpful. A viable next step would be to use guess and check. Since Eric climbed the fewest mountains, let us assume he climbed just one by himself. Say, $c = 1$. Then, his total would be six mountains. Since Kim climbed twice as much as him, her total would then be 12. This would mean $b = 2$. If Ryan climbed three times as much as Kim, then his total is 36 (which is also six times as much as Eric) and, thus, a would have to be 29, which is answer option c.

Here is the Venn diagram of the solution:



Let's consider whether or not the other answer options are viable:
If we let $c = 2$, then Eric's total would be 7, Kim's total would be 14, and Ryan's total 42. Then, $b = 4$ and $a = 33$, which isn't an answer option.
If we let $c = 3$, then Eric's total would be 8, Kim's total would be 16, and Ryan's total 48. Then, $b = 6$ and $a = 37$, which isn't an answer option.
If we let $c = 4$, then Eric's total would be 9, Kim's total would be 18, and Ryan's total 54. Then, $b = 8$ and $a = 41$, which isn't an answer option.
If we let $c = 5$, then Eric's total would be 10, Kim's total would be 20, and Ryan's total 60. Then, $b = 10$ and $a = 45$, which isn't an answer option.
If we let $c = 6$, then Eric's total would be 11, Kim's total would be 22, and Ryan's total 66. Then, $b = 12$ and $a = 49$, which isn't an answer option.
If we let $c = 7$, then Eric's total would be 12, Kim's total would be 24, and Ryan's total 72. Then, $b = 14$ and $a = 53$, which isn't an answer option.
If we let $c = 8$, then Eric's total would be 13, Kim's total would be 26, and Ryan's total 78. Then, $b = 16$ and $a = 57$, which isn't an answer option.
We don't need to check higher values for c because no answer option is higher than 57.

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11. Find the sum of the first 100 terms for the following sequence of numbers 35, 37, 39, …

a) 13,400 b) 27,000 c) 13,500

d) 26,800 e) 13,600

Correct answer: a) 13,400.
Solution: First, find the 100th term by using $a_n = a_1 + (n-1)d$, which in this case is $a_{100} = 35 + [(100) - 1](2) = 35 + 99 \cdot 2 = 35 + 198 = 233$. Then the sum is $S_n = \frac{n(a_1 + a_n)}{2}$ which in this case is $S_n = \frac{(100)[(35)+(233)]}{2} = \frac{100 \cdot 268}{2} = \frac{26{,}800}{2} = 13,400$.

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12. What is the smallest integer that is one-half of a square and one-third of a cube?

a) 64 b) 1728 c) 72

d) 36 e) 2

Correct answer: c) 72.
Solution: We want a number that when multiplied by 2 yields a square and when multiplied by 3 yields a cube. We want the smallest such number, so use powers of 2 and 3. The power of 2 should be odd and a multiple of 3, and the power of 3 should be even

and one less than a multiple of 3. Therefore, $2^3 \cdot 3^2$ is the smallest number that meets these criteria.

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13. If you divide John's age by 5, its remainder is 1. And if you divide John's age by 9, its remainder is 7. How old is Johnny today, if he had a secondary school math exam yesterday, on his birthday?

a) 12 b) 14 c) 16

d) 18 e) 20

Correct answer: c) 16.
Solution: The integers which produce the remainder 1 when divided by 5 are: 1, 6, 11, **16**, 21, 26, 31, 36, ... And the integers which produce the remainder 7 when divided by 9 are: 7, **16**, 25, 34, 43, 52, …

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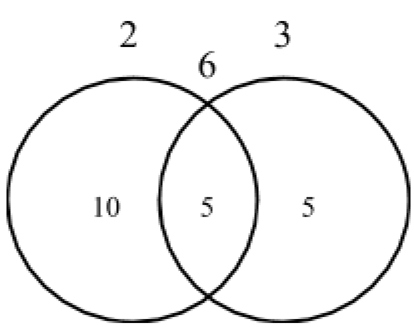
14. How many positive whole numbers (or positive integers) less than or equal to 30 have either 2 or 3 as a factor?

a) 5 b) 10 c) 15

d) 20 e) 25

Correct answer: d) 20.
Solution:
$30 \div 6 = 5$
$30 \div 2 = 15$
$30 \div 3 = 10$



Total is 20

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15. The length of all three sides of a triangle are integer numbers. The length of one side is 6 and the length of the second side is 1. What is the length of the third side of the triangle?

a) 1　　b) 3　　c) 5

d) 6　　e) 7

Correct answer: d) 6.
Solution: The length of the third side of the triangle is 6. Since the sum of lengths of any two sides of a triangle must exceed the length of the third one, no other choice of given answers will satisfy this condition, but $1 + 1 < 6$, $1 + 3 < 6$, $1 + 5 = 6$, and $1 + 6 = 7$.

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16. What is the difference between the sum of the first 500 positive even numbers and the sum of the first 500 positive odd numbers?

a) 5　　b) 10　　c) 100

d) 500　　e) 1000

Correct answer: d) 500.
Solution: $(2 + 4 + 6 + 8 + 10 + \ldots + 1000) - (1 + 3 + 5 + 7 + 9 + \ldots + 999) = (2 - 1) + (4 - 3) + (6 - 5) + (8 - 7) + (10 - 9) + \ldots + (1000 - 999) = 1 + 1 + 1 + 1 + 1 + \ldots + 1 = 500$.

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17. If $f(x-1) = (1-x)(x+2)(x-3)$, on which intervals is the function $f(x)$ negative?

a) $(-2,\ 1)$　　b) $(-2,\ 1) \cup (2,\ \infty)$　　c) $(-\infty,\ -3) \cup (-2,\ 1)$

d) $(-3,\ 0) \cup (2, \infty)$　　e) None of the above

Correct answer: d) $(-3,\ 0) \cup (2, \infty)$.
Solution: Let $y = x - 1$, then $f(y) = -y(y+3)(y-2)$

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18. Solve $log_4 (x^2 - 9) - log_4 (x+3) = 3$

a) -67　　b) 67　　c) 3

d) -3　　e) None of the above.

Correct answer: b) 67.

Solution:

$log_4(x^2 - 9) - log_4(x + 3) = 3$

$log_4\left(\frac{x^2-9}{x+3}\right) = 3$

$log_4\left[\frac{(x+3)(x-3)}{x+3}\right] = 3$

$log_4(x - 3) = 3$

$4^3 = x - 3$

$64 = x - 3$

$x = 67$

Check:

$log_4[(67)^2 - 9] - log_4[(67) + 3]\ \ ? =\ 3$

$log_4[4489 - 9] - log_4[(67) + 3]\ \ ? = 3$

$log_4[4480] - log_4[70]\ \ ? =\ 3$

$log_4\left(\frac{4480}{70}\right)\ \ ? =\ 3$

$log_4(64)\ \ ? =\ 3$

$4^3 = 64\ \ \surd$

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19. Find the value of $\frac{a}{b}$ if $\frac{a}{b} = \frac{a+48}{b+6}$.

a) 24 b) 144 c) 8

d) 6 e) 48

Correct answer: c) 8.

Solution: $\frac{a}{b} = \frac{a+48}{b+6} \rightarrow ab + 6a = ab + 48b \rightarrow a = 8b \rightarrow \frac{a}{b} = 8$

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20. A family with four children has a girl named Nancy. What is the probability that all four children are girls, if the probability of having a boy is $\frac{1}{2}$ and the probability of having a girl is $\frac{1}{2}$?

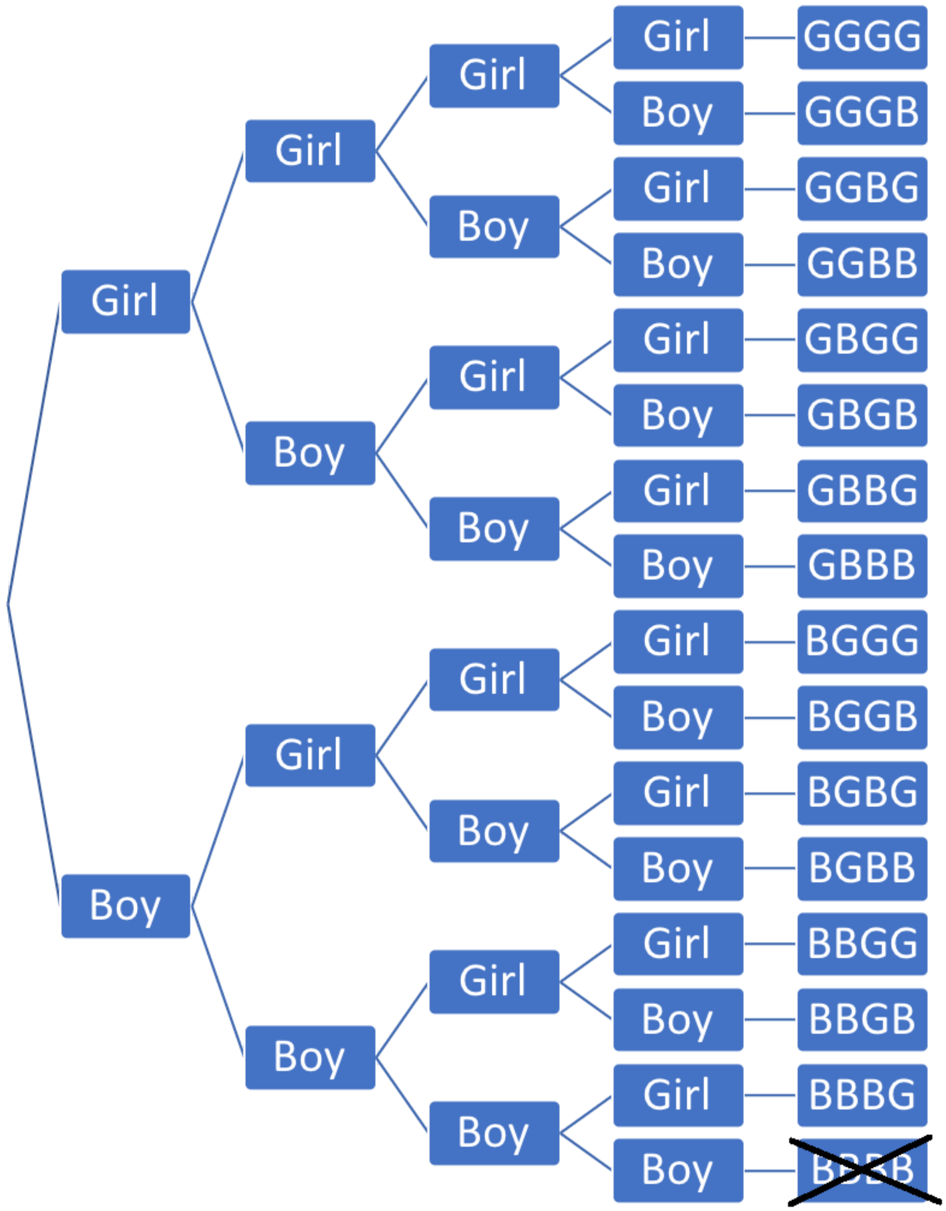
a) 1 b) $\frac{1}{2}$ c) $\frac{1}{16}$

d) $\frac{1}{15}$ e) $\frac{1}{4}$

Correct answer: d) $\frac{1}{15}$.

Solution: There are $2^4 = 16$ different families with 4 children, all equally likely. Since one of the children is Nancy, i.e., a girl, the family cannot have all four boys. Hence,

there are total of 16 – 1 possible families with 4 children, and only one with all four girls. Thus, $\frac{1}{15}$.
One can also use a tree diagram:



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21. Find x if $25^{(x+1)} = 625^{2x}$.

a) $x = \frac{1}{4}$ b) $x = \frac{1}{3}$ c) $x = \frac{1}{2}$

d) $x = \frac{2}{3}$ e) $x = \frac{3}{4}$

Correct answer: b) $x = \frac{1}{3}$.

Solution: Express both sides with the same base of 5: $5^{2(x+1)} = 5^{4(2x)}$. Set the exponents equal to each other and solve for x: $2(x+1) = 4(2x)$. The value for x which will make the original equation true is $x = \frac{1}{3}$.

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22. On level ground, a 10-foot pole is a certain distance from a 15-foot pole. If lines are drawn from the top of each pole to the bottom of the other pole as shown in the drawing, the lines intersect at a point 6 feet above the ground. What is the distance between the poles?

a) 12.5 feet   b) 5 feet   c) 25 feet

d) None of the above are possible.   e) Any of the above are possible.

Correct answer: e) Any of the above are possible.

Solution: Suppose w is the distance between the bases of the poles. Assign coordinates so the top of the 10 foot pole is at (0, 10) and the top of the 15 foot pole is at $(w,\ 15)$. Then the equations of the lines are $y = \frac{15}{w}x$ and $y = 10 - \frac{10}{w}x$. Setting these equations equal to each other yields $x = \frac{10w}{25}$, and plugging this value into either equation yields $y = 6$. In other words, the height of crossing is 6, regardless of the distance w.

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23. Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

a) 30%   b) 40%   c) 50%

d) 60%   e) 70%

Correct answer: a) 30%.

Solution: Let j represent how far Jerry walked, and s represent how far Sylvia walked. Since the field is a square, and Jerry walked two sides of it, while Silvia walked the diagonal, we can simply define the side of the square field to be 1, and find the distances they walked. Since Jerry walked two sides, $j = 2$. Since Silvia walked the diagonal, she walked the hypotenuse of a 45°-45°-90° triangle with leg length 1. Thus, $s = 1.414...$ We can then take $\frac{j-s}{j} = \frac{2-1.414...}{2} = 0.29289...$, which implies (a).

24. Find the volume of a cone whose base has an area of two square-units, and height of six units.

a) 4 b) 6 c) 2

d) 2.33 e) 4.33

Correct answer: a) 4.
Solution: The volume of a cone is given by $v = \pi r^2 \frac{h}{3} = 2\frac{6}{3} = 4$.

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25. Find the smallest positive integer that is divisible by exactly 11 positive integers.

a) 72 b) 108 c) 576

d) 500 e) 1024

Correct answer: e) 1024.
Solution: The number of positive divisors of an integer boils down to its prime factorization. For example, the number of divisors of $p^a q^b$ (with p, q prime) would be $(a+1)(b+1)$ since the divisors have the form $p^i q^j$, where $0 \le i \le a,\ 0 \le j \le b$. Since 11 is prime, the only way to have exactly 11 positive divisors is with a number of the form p^{10}. The smallest prime p is 2, so the answer is $2^{10} = 1024$.

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26. If $a > 0$, then the quadratic function $y = x^2 + ax + a$ has

a) One zero b) Two zeros c) No zeros

d) Any of the above are possible.

e) None of the above are possible.

Correct answer: d) Any of the above are possible.
Solution: Let $\Delta = a(a-4)$ can be positive, negative, or zero for $a > 0$.

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27. A rectangle is partitioned into 4 subrectangles as shown below. If the subrectangles have the indicated areas, find the area of the unknown rectangle.

210	240
91	?

a) 78 b) 98 c) 104

d) 270 e) 390

Correct answer: c) 104.
Solution:

$30 \times 7 = 210$	$30 \times 8 = 240$
$13 \times 7 = 91$	$13 \times 8 = 104$

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28. The total number of passengers riding a certain city bus during the during the day is 1,000. If the child's fare is \$.50, the adult fare is \$1, and the senior fare is \$.50, and the total revenue from the day is \$600. It is also known that an equal number of seniors ride the bus as children. Use this information to determine total number of children and adults that ride the bus.

a) 600 b) 200 c) 300

d) 133 e) 400

Correct answer: a) 600.
Solution: Let x represent the number of children, y the number of adults, and z the number of seniors that ride the bus. The information corresponds to the following equations:

$$x + y + z = 1,000$$
$$0.5x + y + 0.5z = 600$$
$$x = z$$

Using the last relationship yields:

$$2x + y = 1,000$$

$$x + y = 600$$

Hence $x = 400$, $y = 200$, and the total is 600.

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29. You and two of your friends go to a restaurant for lunch. At the end of the meal, all three of you flip a fair coin, getting either a head or a tail. If two of the three coin flip outcomes match, the person with the non-matching outcome pays for everyone's lunch. If all three coin flip outcomes are the same, each person pays for their own lunch. What is the probability that you pay for everyone's lunch?

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$

d) $\frac{1}{6}$ e) $\frac{1}{8}$

Correct answer: c) 1/4.
Solution: There are 8 possible, equally likely outcomes of the three coin flips. There are two cases where you pay for lunch: either you get a head and your friends get tails or you get a tail and your friends get heads.

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30. A rain gutter is to be made of aluminum sheets that are 12 inches wide by turning up the edges 90°. What depth will provide maximal cross-sectional area and hence allow the largest water flow?

a) 4.5 inches b) 2 inches c) 3.5 inches

d) 4 inches e) 3 inches

Correct answer: e) 3 inches.
Solution: The cross-sectional area of the gutter is given by $(12 - 2x)x$, where x is the distance in inches from the edge of the sheet at which the bend is made. Since $y = (12 - 2x)x = 12x - 2x^2$ is a quadratic equation with a negative leading coefficient, its graph is a parabola which opens down. The largest y-value lies at the vertex, whose x-coordinate is $\frac{-b}{2a} = \frac{-12}{-4} = 3$ inches.

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31. A group of friends took a bus trip. Each traveler gave the bus driver a tip using the same nine coins. The total tip was \$8.41. How many dimes did the driver receive?

a) 0 b) 9 c) 18

d) 27 e) 36

Correct Correct answer: a) 0.
Solution: We know that all travelers gave the same tip amount. Factors of 841 are 1, 29, and 841. We are told it was a group of friends, implying that there was more than 1 traveler. So the number of friends must be 29 or 841. If it was 841, then each person gave only 1 cent, so there must have been 29 friends, each of whom gave a 29 cent tip. The only way to make 29 cents with nine coins is with ve nickels and four pennies, so no dimes were given.

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32. Seven friends took a quiz. Each got a score that is a whole number between 1 and 100. No two friends got the same score. The median score received by the friends was 50 and the range (the maximum score minus the minimum score) was 20. What is the highest score that any of the friends could have received?

a) 50 b) 53 c) 60

d) 67 e) 99

Correct answer: d) 67.
Solution: The 4th highest score must be 50, the median. Since the range is fixed at 20, to maximize the highest score, the lowest score must be maximized. The highest values that the three lowest performing friends could get would be 49, 48, 47. With the lowest value at 47, the highest value must be 67.

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33. Pablo buys popsicles for his friends. The store sells single popsicles for \$1 each, 3-popsicle boxes for \$2 each, and 5-popsicle boxes for \$3. What is the greatest number of popsicles that Pablo can buy with \$8?

a) 8 b) 11 c) 12

d) 13 e) 15

Correct answer: d) 13 popsicles.
Solution: \$3 boxes give us the most popsicles/dollar, so we want to buy as many of those as possible. After buying 2, we have \$2 left. We cannot buy a third \$3 box, so we opt for the \$2 box instead (since it has a higher popsicles/dollar ratio than the \$1 pack). We're now out of money. We bought \$5 + \$5 + \$3 = \$13 popsicles, so the answer is d) 13.

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34. Find the last digit of 3^{999}.

a) 1 b) 3 c) 7

d) 9 e) None of the above are possible.

Correct answer: c) 7.
Solution: Look for a pattern:
$3^1 = \underline{3}$
$3^2 = \underline{9}$
$3^3 = 2\underline{7}$
$3^4 = 8\underline{1}$
$3^5 = 24\underline{3}$
$3^6 = 72\underline{9}$
$3^7 = 218\underline{7}$
$3^8 = 656\underline{1}$
The pattern repeats every four numbers: 3, 9, 7, 1. So, $999 \div 4 = 249$ R3. Thus, the last digit is 7.

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35. What is the domain of the function $f(x) = (x - 3x^2)^{\frac{1}{2}}$?

a) $\left[0, \frac{1}{3}\right]$ b) $(-\infty, \frac{1}{3}]$ c) $(-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$

d) $\left(0, \frac{1}{3}\right)$ e) $(-\infty, \infty)$

Correct answer: a) $\left[0, \frac{1}{3}\right]$.
Solution: We want $x - 3x^2 \geq 0$. The graph of $f(x) = x - 3x^2 = x(1 - 3x)$ is a parabola which opens down. The x-intercepts of f are $x = 0$ and $x = \frac{1}{3}$. It follows that $f(x) \geq 0$ on the interval $\left[0, \frac{1}{3}\right]$.

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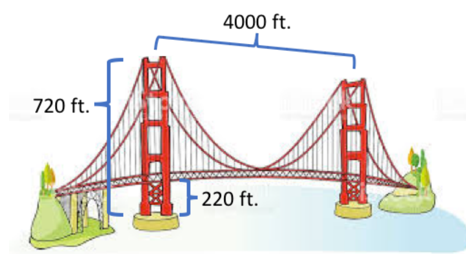
36. The Golden Gate Bridge is a suspension bridge which spans the entrance to San Francisco Bay. Its 720 foot tall towers are 4000 feet apart. The bridge is suspended from two huge cables. The roadway is 220 feet above the base of the towers. The cables are parabolic in shape and touch the road surface at the center of the bridge. Find the height of the cable above the road at a distance of 1000 feet from the center of the bridge.



a) 360 feet   b) 250 feet   c) 150 feet

d) 125 feet   e) 180 feet

Correct answer: d) 125 feet
Solution: Take as our coordinate axes the roadway and the line perpendicular to the midpoint of the bridge. Then the point (2000, 500) lies on the parabola $y = ax^2$ which describes the cable's shape. Thus, $500 = a(2000)^2$, so that $a = \frac{500}{2000^2}$. We want y when $x = 1000$, or $y = \frac{(500)(1000)^2}{(2000)^2} = \frac{500}{4} = 125$ feet.

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37. Find all the values of x which satisfy the inequality $\sqrt{(x-2)^2} < |x|$.

a) $(-2, 0)$ b) $(-2,\ 1) \cup (2,\ \infty)$ c) $(1, \infty)$

d) $(2, 5)$ e) None of the above

Answer: c) $(1, \infty)$.
Solution: With $|x-2| < |x|$, we have three possible cases. Case 1: $x < 0$ gives no solution. Case 2: $0 \le x \le 2$ gives a solution of $(1, 2]$. Case 3: $x > 0$ gives a solution of $(2,\ \infty)$.

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38. $(3\sqrt{7}+4)^3 + (3\sqrt{7}-4)^3 =$

a) 1   b) $3\sqrt{7}$ c) $21\sqrt{7}$

d) $111\sqrt{7}$ e) $666\sqrt{7}$

Correct answer: e) $666\sqrt{7}$
Solution: Let $a = 3\sqrt{7}+4$ and $b = 3\sqrt{7}-4$.
Then we have a^3+b^3. Factor sum of cubes to be $(a+b)\left(a^2-ab+b^2\right)$.
Find $a+b$ to be $\left(3\sqrt{7}+4\right)+\left(3\sqrt{7}-4\right) = 6\sqrt{7}$.

Find a^2 to be $(3\sqrt{7}+4)(3\sqrt{7}+4) = 79+24\sqrt{7}$.
Find ab to be $(3\sqrt{7}+4)(3\sqrt{7}-4) = 47$.
Find b^2 to be $(3\sqrt{7}-4)(3\sqrt{7}-4) = 79-24\sqrt{7}$.
Then,

$$\begin{aligned}(a+b)(a^2-ab+b^2) &= (6\sqrt{7})[(79+24\sqrt{7})-(47)+(79-24\sqrt{7})] \\ &= 6\sqrt{7}(79+24\sqrt{7}-47+79-24\sqrt{7}) \\ &= 6\sqrt{7}(158-47) \\ &= 6\sqrt{7}\cdot 111 \\ &= 666\sqrt{7}\end{aligned}$$

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39. Let x and y have the following relationship:

$$3xy^2 - 3yx^2 = 6$$
$$x^3 - y^3 =- 7$$

Determine the value of $(x-y)^3 =$

a) –1 b) 1 c) 0

d) –8 e) 27

Correct answer: a) –1.
Solution: Use the binomial theorem $(x+y)^3 = x^3 + 3xy^2 - 3yx^2 - y^3 = 6 - 7 =- 1$

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40. Find the difference in area of the two circular discs, $(x+3)^2 + (y-6)^2 = 4$ and $(x+3)^2 + (y-5)^2 = 9$.

a) 4π b) 5π c) 7π

d) 9π e) 11π

Correct answer: b) 5π.
Solution: The first equation is a circular disc with radius of 3 and the second equation is a circular disc with a radius of 2. So, $\pi(3)^2 - \pi(2)^2 = 9\pi - 4\pi = 5\pi$.
