

Name: _____

State Math Contest – Junior Exam

Instructions:

- Do not turn this page until your proctor tells you.
 - Enter your name, grade, and school information following the instructions given by your proctor.
 - Calculators are **not** allowed on this exam.
 - This is a multiple choice test with 40 questions. Each question is followed by answers marked a), b), c), d), and e). Only one answer is correct.
 - Mark your answer to each problem on the bubble sheet Answer Form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.
 - **Scoring:** You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
 - You will have 2 hours and 30 minutes to finish the test.
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13. High schools in Utah are divided into different classifications, with the largest schools classified as 5A and the next largest schools classified as 4A.
- If the smallest 5A school is reclassified as 4A, what will happen to the average size of schools in the two classifications?
- a) The average 4A size will go down, and the average 5A size will go up.
 - b) The average 4A size will go up, and the average 5A size will go up.
 - c) The average 4A size will go down, and the average 5A size will go down.
 - d) The average 4A size will go up, and the average 5A size will go down.
 - e) None of the above is always true.
14. How many ways can you write 5 as the sum of one or more positive integers if different orders are not counted differently? For example, there are three ways to write 3 in this way: $1 + 1 + 1$, $1 + 2$, and 3.
- a) 7
 - b) 6
 - c) 8
 - d) 5
 - e) 10
15. How many real solutions does the equation $x^{3/2} - 32x^{1/2} = 0$ have?
- a) 0
 - b) 1
 - c) 2
 - d) 3
 - e) 4
16. When slicing a rectangular cake, what is the smallest number of straight cuts that you need to make exactly 7 pieces?
- a) 7
 - b) 6
 - c) 5
 - d) 4
 - e) 3
17. A very thin disk has an area (on one side) of 4π . A square window is cut into a wall. What is the smallest area (the *lower bound*) the window can have and still be large enough for the disk to fit through?
- a) $16/\pi$
 - b) 4π
 - c) $8\sqrt{2}$
 - d) 8
 - e) 16

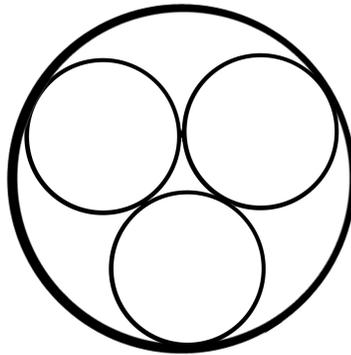
31. For a certain baseball team the probability of winning any game is P , (the probability of winning a particular game is independent of any other games). What is the probability the team wins 3 out of 5 games?

- a) $10P^2(1 - P)^3$ b) $10P^3(1 - P)^2$ c) $5P^3(1 - P)^2$
d) $5P^2(1 - P)^3$ e) $P^3(1 - P)^2$

32. If x is the *fraction* of numbers between 1 and 1,000, inclusive, which contain 4 as a digit, and y is the *fraction* of numbers between 1 and 10,000, inclusive which contain 4 as a digit, what is x/y ?

- a) $2/3$ b) $3/4$ c) $27/34$
d) $271/3439$ e) $2710/3439$

33. Given that the area of the outer circle is ten square units, find the area of any one of the three equal circles which are tangent to each other and to the outer circle, and inscribed inside the circle of ten square units.



- a) $30(7 - 4\sqrt{3})$ square units. b) 2.5 square units.
c) $\frac{10}{(3 + \sqrt{2})}$ square units. d) $\sqrt[3]{10}$ square units.
e) 2 square units.

34. Let $f(x) = 9x^2 + dx + 4$. For certain values of d , the equation $f(x) = 0$ has only one solution. For such a value of d , which value of x could be a solution to $f(x) = 0$?

- a) $\frac{2}{3}$ b) 1 c) $\frac{4}{3}$
d) 3 e) 12

35. The natives of Wee-jee Islands rate 2 spears as worth 3 fishhooks and a knife, and will give 25 coconuts for 3 spears, 2 knives, and a fishhook together. Assuming each item is worth a whole number of coconuts, how many coconuts will the natives give for each article separately?

	Item	Worth in Coconuts
a)	fishhook	1
	knife	3
	spear	3

	Item	Worth in Coconuts
b)	fishhook	2
	knife	2
	spear	4

	Item	Worth in Coconuts
c)	fishhook	1
	knife	5
	spear	4

	Item	Worth in Coconuts
d)	fishhook	3
	knife	3
	spear	6

	Item	Worth in Coconuts
e)	fishhook	2
	knife	4
	spear	5

36. An octagon in the plane is symmetric about the x -axis, the y -axis, and the line whose equation is $y = x$. If $(1, \sqrt{3})$ is a vertex of the octagon, find its area.

a) $6\sqrt{3}$

b) 11

c) $6 + 2\sqrt{3}$

d) $2 + 6\sqrt{3}$

e) $4 + 4\sqrt{3}$

37. Five points are placed in a square with side length 1. What is the largest distance d so that every pair of points is at least d apart from each other?

a) 1

b) $\sqrt{2}$

c) $\sqrt{3}/2$

d) $1/2$

e) $\sqrt{2}/2$

38. A square number is an integer number which is the square of another integer.

Positive square numbers satisfy the following properties:

- The units digit of a square number can only be 0, 1, 4, 5, 6, or 9.
- The digital root of a square number can only be 1, 4, 7, or 9.

The *digital root* is found by adding the digits of the number. If you get more than one digit you add the digits of the new number. Continue this until you get to a single digit. This digit is the digital root.

One of the following numbers is a square. Which one is it?

- a) 4, 751, 006, 864, 295, 101
- b) 3, 669, 517, 136, 205, 224
- c) 2, 512, 339, 789, 576, 516
- d) 1, 898, 732, 825, 398, 318
- e) 5, 901, 643, 220, 186, 107

39. A regular octahedron is formed by setting its vertices at the centers of the faces of the cube. Another regular octahedron is formed around the cube by making the center of each triangle of the octahedron hit at a vertex of the cube. What is the ratio of the volume of the larger octahedron to that of the smaller octahedron?

- a) $2\sqrt{2}$
- b) $27/8$
- c) $3\sqrt{3}$
- d) 8
- e) 27

40. In $\triangle ABC$, $AC = 13$, $BC = 15$ and the area of $\triangle ABC = 84$. If $CD = 7$, $CE = 13$, and the area of $\triangle CDE$ can be represented as $\frac{p}{q}$ where p and q are relatively prime positive integers, find q .

- a) 3
- b) 5
- c) 7
- d) 11
- e) 13

