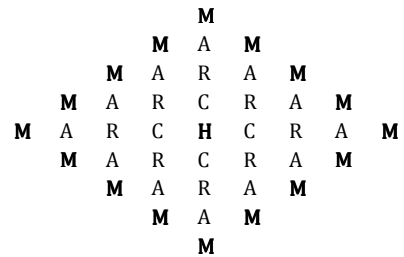


Utah State Mathematics Contest  
Junior Exam  
March 16, 2011

1. Making only vertical or horizontal moves between adjacent letters, in how many ways can you start from an 'M' and move to the 'H', spelling the word 'MARCH' ?

- (a) 64                      (b) 112                      (c) 120  
(d) 60                      (e) 124



2. If the radius of a circle is increased by 200%, then the area will be increased by:

- (a) 800%      (b) 200%      (c) 900%      (d) 300%      (e) 400%

3. Assuming  $m \neq n, m \neq 0, n \neq 0$ ,  $\frac{mn - m^2}{mn - n^2} - \frac{n^2 - m^2}{mn}$  reduced to lowest terms is equal to:

- (a) 1              (b)  $\frac{m}{n}$               (c)  $\frac{2m^2 - n^2}{n}$               (d)  $\frac{2m^2 - n^2}{m}$               (e)  $-\frac{n}{m}$

4. Which of the following is a perfect square?

- (a)  $4! \cdot 9!$       (b)  $9! \cdot 10!$       (c)  $8! \cdot 10!$       (d)  $8! \cdot 9!$       (e)  $8! \cdot 9! \cdot 10!$

5. How many distinct lines make up the altitudes, medians and angle bisectors in a triangle that is isosceles but not equilateral?

- (a) 3              (b) 7              (c) 9              (d) 5              (e) 6

6. If eight ducks eat eight bushels of oats in eight days, and if twelve geese eat twelve bushels of oats in twelve days, then approximately how many bushels of oats (to the nearest bushel) will twenty ducks and twenty geese eat in twenty days?

- (a) 65              (b) 57              (c) 83              (d) 40              (e) 106

7. Each day, for three days, Randall withdraws exactly 20% of the money that was in his sock drawer at the start of the day. At the end of the third day, he had \$30.72. How much money was in the sock drawer originally?

- (a) \$76.80      (b) \$67.50      (c) \$60.00      (d) \$49.16      (e) \$48.00

8. Dante wants to buy four donuts from an ample supply of three types of donuts: glazed, chocolate, and powdered. How many different selections are possible?

- (a) 6            (b) 9            (c) 12            (d) 15            (e) 18

9. The equation  $\frac{x+8}{5} = \frac{4x^2-48x}{x^2-12x}$  has how many distinct solutions?

- (a) 0            (b) 1            (c) 2            (d) 3            (e) infinitely many

10. Assuming each of the following choices of successive discounts is available, which of the following choices is most advantageous to the consumer?

- (a) two 15% discounts            (b) five 6% discounts            (c) three 10% discounts  
(d) six 5% discounts            (e) one 30% discount

11. What is the largest number of acute angles that can be found in a convex decagon?

- (a) 2            (b) 3            (c) 4            (d) 5            (e) 6

12. A large group of puppies and kittens are waiting to be adopted. If 15 kittens are adopted, the ratio of puppies to kittens is now 3:1. Later, 24 puppies are adopted, putting the ratio of puppies to kittens at 1:3. How many animals were there originally?

- (a) 51            (b) 117            (c) 99            (d) 85            (e) 78

13. Given a circle and two parallel lines both tangent to the circle, how many points are equidistant from all three objects? (assume all objects are contained in a plane)

- (a) 1            (b) 2            (c) 3            (d) 4            (e) 5

14. How many of the following statements *must* be true for all real numbers  $a$  and  $b$  (where  $a \neq b$  and  $a \neq -b$ )?

(i)  $\frac{a}{(a-b)} < \frac{b}{(a-b)}$

(ii)  $\frac{a}{(a+b)} < \frac{b}{(a+b)}$

(iii)  $a(a-b) > b(a-b)$

(iv)  $a(a+b) > b(a+b)$

- (a) 0            (b) 1            (c) 2            (d) 3            (e) 4

15. A regular hexagon has one circle inscribed within and another circle circumscribed about it. What is the ratio of the area of the smaller circle to the area of the larger?

- (a)  $\frac{\sqrt{2}}{2}$       (b)  $\frac{\sqrt{3}}{2}$       (c)  $\frac{1}{2}$       (d)  $\frac{\sqrt{3}}{4}$       (e)  $\frac{3}{4}$

16. For how many positive integers  $n$  does the sum  $1 + 2 + \dots + n$  evenly divide  $6n$ ?

- (a) 11      (b) 9      (c) 7      (d) 5      (e) 3

17. The top of a 25-foot ladder leans against the side of the school, with the base of the ladder 7 feet from the building. If the top of the ladder were to slide 4 feet down, the base of the ladder would slide:

- (a) 8 ft      (b) 10 ft      (c) 5 ft      (d) 7 ft      (e) 4 ft

18. There is a positive integer,  $B$ , such that  $2B$  is a perfect square,  $3B$  is a perfect cube, and  $5B$  is a perfect fifth (an integer to the fifth power). Assume that  $B$  is the smallest such number. How many factors are in the prime factorization of  $B$ ?

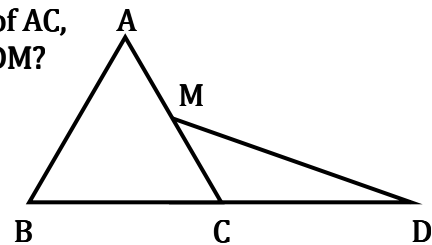
- (a) 59      (b) 7      (c) 29      (d) 31      (e) 61

19. The number of terms in the expanded form of  $[(x - 2y)^2(x + 2y)^2]^2$  when simplified is:

- (a) 4      (b) 7      (c) 8      (d) 9      (e) 5

20. Equilateral  $\triangle ABC$  has side length 6.  $M$  is the midpoint of  $AC$ , and  $C$  is the midpoint of  $BD$ . What is the area of  $\triangle CDM$ ?

- (a)  $\frac{9\sqrt{2}}{2}$       (b)  $\frac{27}{4}$       (c)  $\frac{9\sqrt{3}}{2}$       (d) 8      (e)  $9\sqrt{2}$



21. On a recent algebra exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the difference between the mean and the median score on the exam?

- (a) 0      (b) 1      (c) 2      (d) 3      (e) 4

22. There are two values of  $p$  for which the equation  $3x^2 + px - 5x + 8 = 0$  has exactly one distinct solution. What is the sum of those values?

- (a) 3      (b) 7      (c) 4      (d) 6      (e) 10

23. The areas (in square inches) of the front, top, and side of a rectangular box are 3, 10, and 12, respectively. Which of the following choices is the closest to the measure of the volume of the box in cubic inches?

- (a) 22      (b) 27      (c) 19      (d) 16      (e) 25

24. In which of the following cases are  $x$  and  $y$  neither directly nor inversely proportional?

- (a)  $\frac{2x}{5y} = 30$       (b)  $2x - y = 8$       (c)  $4xy = \sqrt{11}$   
(d)  $7x = 24y$       (e)  $3x + 5y = 0$

25. The expression  $\sqrt{5 + 2\sqrt{6}} - \sqrt{5 - 2\sqrt{6}}$  is equivalent to which of the following?

- (a)  $2\sqrt{6}$       (b)  $2\sqrt{2}$       (c)  $3\sqrt{5}$       (d)  $3\sqrt{3}$       (e)  $5\sqrt{2}$

26. Two trains are travelling on perpendicular tracks which pass through the same Union Station. The Pacific Railway is travelling north, while the Atlantic Zephyr is travelling east. As the Railway passes through Union Station, the Zephyr is 25 miles west of the station. After 20 minutes, the two trains are the same distance from the station. Another 80 minutes later, the two trains are again the same distance from the station. What is the ratio of the speed of the Zephyr to the speed of the Railway?

- (a)  $\frac{8}{5}$       (b)  $\frac{4}{3}$       (c)  $\frac{7}{5}$       (d)  $\frac{3}{2}$       (e)  $\frac{5}{3}$

27. The number of solution pairs that are positive integers to the linear equation  $3x + 16y = 2011$  is:

- (a) 38      (b) 39      (c) 40      (d) 41      (e) 42

28. Let  $x$  and  $y$  be two-digit integers such that  $y$  is obtained by reversing the digits of  $x$ . Suppose that the integers  $x$  and  $y$  also satisfy the equation  $x^2 - y^2 = z^2$  for some positive integer  $z$ . What is the value of  $x + y + z$ ?

- (a) 112      (b) 154      (c) 88      (d) 116      (e) 144

29. If the diagonals of a quadrilateral are perpendicular to one another, within which of the following classifications would the shape necessarily fall?

- (a) a square                      (b) a rhombus                      (c) an isosceles trapezoid  
(d) a rectangle                      (e) none of these choices

30. Concrete sections on the new portions of I-15 are 80 feet long. As a car drives over the seams where sections meet, a brief thump is heard by the passengers. The speed of the car in miles per hour is approximately equal to the number of thumps heard in how many seconds?

- (a) 40              (b) 85              (c) 45              (d) 55              (e) 70

31. Each student in a group of 80 students is right- or left-handed and right- or left-eyed (primary eye used for focusing). If 45 students are right-handed-right-eyed, 24 students are left-eyed, and 15 students are left-handed, then the number of left-handed-left-eyed students is:

- (a) 3              (b) 4              (c) 0              (d) 6              (e) 2

32. Two numbers exist such that their difference, sum and product are to each other as 3:11:42. What is the larger of the two numbers?

- (a) 16              (b) 6              (c)  $\frac{8}{3}$               (d) 8              (e)  $\frac{21}{2}$

33. For how many integer values of  $b$  can the expression  $6x^2 + bx + 6$  be factored into precisely two prime binomial factors?

- (a) 0              (b) 2              (c) 4              (d) 6              (e) 8

34. Millie buys a large number of hamburgers at a rate of 3 for \$2 and the same number of corndogs at 5 for \$4. If she sells all of the hamburgers and corndogs at a single rate, what price must she use to break even?

- (a) 6 for \$4.40                      (b) 8 for \$6.00                      (c) 10 for \$7.25  
(d) 9 for \$6.80                      (e) 4 for \$3.20

35. How many sets of two or more consecutive positive integers have a sum of 21?  
(a) 1      (b) 2      (c) 3      (d) 4      (e) 5
36. The graphs of the equations  $x^2 + 2y = 16$  and  $x + y = 8$  have two points of intersection. What is the distance between these two points?  
(a)  $2\sqrt{2}$       (b)  $\sqrt{5}$       (c)  $2\sqrt{3}$       (d)  $2\sqrt{5}$       (e) 2
37. A barrel contains a selection of colored cubes, each of which is yellow, blue, or green. The number of green cubes is at least half of the number of blue cubes, and at most one third of the number of yellow cubes. The cubes which are blue or green number at least 34. The minimum number of yellow cubes is:  
(a) 61      (b) 27      (c) 36      (d) 44      (e) 52
38. Imagine two circles, the larger with center  $M$  and radius  $m$ , the smaller with center  $N$  and radius  $n$ . Constructing line segment  $MN$ , which of the following choices cannot be true of the length of the line segment connecting the two centers?  
(a) It is equal to  $m - n$       (b) It is less than  $m - n$       (c) It is equal to  $m + n$   
(d) It is less than  $m + n$       (e) Any of these may be true
39. A parabola with the equation  $y = x^2 + bx + c$  passes through the points  $(1, 4)$  and  $(5, 4)$ . What is the value of  $c$ ?  
(a) 9      (b) 0      (c) -2      (d) 5      (e) -4
40. Snowden left his entire estate to his four war buddies. He left half to Appleby and Huple, who shared their portion at a ratio of 5 to 4. Popinjay received twice as much as Huple. Lastly, Wintergreen received \$4,000. How much did Popinjay receive?  
(a) \$18,000      (b) \$20,000      (c) \$16,000      (d) \$32,000      (e) \$24,000