

Utah State Mathematics Contest
Junior Exam
March 17, 2010

1. In the 2010 Vancouver Olympics, placement in curling semi-finals was based on a 10-team round-robin tournament, in which each of 10 teams plays each other team once, and the top four records move on to the semi-finals. A team is considered to be a strong contender for the gold medal if they win 7 of their 9 matchups. In this system, how many different teams can all end the round-robin tournament with a record of 7 or more wins?
- (a) 2 (b) 3 (c) 4 (d) 5 (e) 6
2. Determine the sum of the reciprocals of all positive integer factors of 28.
- (a) 1 (b) $\frac{27}{14}$ (c) $\frac{3}{2}$ (d) $\frac{55}{28}$ (e) 2
3. Evan has the same birthday as his grandson, James. For the last six consecutive years, Evan's age was a multiple of James'. How old will Evan be at the next time that Evan's age will again be a multiple of his grandson's?
- (a) 54 (b) 64 (c) 70 (d) 72 (e) 80
4. The repeating decimal of form of a number is $0.0\overline{48}$... If such a number were converted to a fraction and reduced to lowest form, what would be the denominator?
- (a) 165 (b) 110 (c) 55 (d) 990 (e) 22
5. What is the sum of all integers, M , for which $M^2 - 8M - 20$ is a prime number?
- (a) 2 (b) 4 (c) 6 (d) 8 (e) 10
6. Yarrrrrggghhhh! Literary critics complain that 80% of all fictional pirate captains have an eye-patch, 75% have a hook-hand, 67% have a peg-leg, and 90% have a pet parrot. By this accounting, at least what percent of these pirate captains must have an eye-patch, a hook-hand, a peg-leg *and* a parrot?
- (a) 7% (b) 12% (c) 18% (d) 26% (e) 36%

7. Let $|U - 10| = V$, given that $U < 10$. What is the value of $U - V$?

- (a) $10 - 2V$ (b) -10 (c) $10 - 2U$ (d) 10 (e) $|U - 10| - U$

8. Havermeyer recently inherited a sizeable sum of money. He paid 30% in taxes and invested 20% of what remained into M&M Enterprises. If M&M received \$11,900 from Havermeyer, how much did Havermeyer pay in taxes?

- (a) \$18,200 (b) \$19,600 (c) \$17,850 (d) \$25,500 (e) \$3,570

9. The midpoints of the three sides of $\triangle ABC$ are $(5, 4)$, $(1, 2)$, and $(-1, 6)$. Which of the following is one of the actual points A, B, or C?

- (a) $(4, 9)$ (b) $(3, 8)$ (c) $(5, 2)$ (d) $(6, 3)$ (e) $(0, 6)$

10. Two tour guides are leading seven tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the condition that each guide must take at least one tourist. How many different groupings of guides and tourists are possible?

- (a) 118 (b) 120 (c) 122 (d) 124 (e) 126

11. The positive integers $C, D, C + D, C - D$ are all prime. The sum of these four primes must be divisible by which of the following?

- (a) 2 (b) 3 (c) 5 (d) 7 (e) The sum is prime

12. Bill randomly selects three different numbers from the set $\{1, 2, 3, 4\}$, while Kerry randomly selects just one number from the set $\{2, 4, 6, 8, 10, 12\}$. What is the probability that Kerry's one number is larger than the sum of Bill's three numbers?

- (a) $\frac{3}{8}$ (b) $\frac{1}{3}$ (c) $\frac{5}{12}$ (d) $\frac{5}{8}$ (e) $\frac{2}{3}$

13. A US military clock reads the time of day to be 2010 hours (this is 8:10 PM). At this time of day, what is the angle formed by the minute and hour hands of a 12-hour clock?

- (a) 160° (b) 165° (c) 170° (d) 172° (e) 175°

14. For a fixed cube, how many unique planes contain at least three of the cube's vertices?

- (a) 6 (b) 12 (c) 14 (d) 20 (e) 28

15. Which of the following equations, if graphed on a Cartesian plane, would have no y-intercept?

(a) $x^2 + y^2 = 2xy$ (b) $|y| + 4 = |x|$ (c) $\sqrt{x} = 6 - \sqrt{y}$

(d) $8^x + 8^y - 65 = 0$ (e) $\frac{x}{10} - \frac{10}{y} = \frac{(x+10)}{10}$

16. Alaskan oil reserves would potentially last for 35 years if used only by the United States.

If the same reserves are also utilized by China, they will last for only 10 years. How many years would this oil last if only used by China?

- (a) 15 (b) 21 (c) 25 (d) 14 (e) 20

17. An amalgam of mercury and cesium, 50 mL in volume, is found to be 30% cesium. A chemist wishes to combine this amalgam with 60 mL of pure cesium, but wants the final mixture to be exactly 60% cesium. To dilute the mixture, the chemist has a large supply of pure mercury. How many milliliters of pure mercury should be combined with the amalgam and pure cesium to produce the desired mixture?

- (a) 0 (b) 5 (c) 10 (d) 15 (e) 20

18. A right triangle is inscribed in a circle with radius equal to $\frac{5}{\sqrt{2}}$. If the lengths of the legs of the triangle are distinct integers, what are these lengths?

- (a) 8 & 4 (b) 9 & 3 (c) 7 & 1 (d) 6 & 5 (e) 11 & 3

19. The mean of three numbers is 7 more than the least of the numbers and 9 less than the greatest. The median of the three numbers is 6. What is their sum?

- (a) 24 (b) 30 (c) 32 (d) 36 (e) 48

20. An arc of 60° on Circle A is half the length of an arc of 90° on Circle B. What is the ratio of the area of Circle A to the area of Circle B?

- (a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{16}{9}$ (d) $\frac{9}{4}$ (e) $\frac{4}{3}$

21. Compare the sum of all positive odd integers less than 2010 and the sum all positive even integers less than 2010. What is the difference between these two sums?

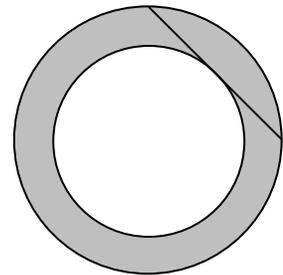
- (a) 0 (b) 1005 (c) 2010 (d) 3015 (e) 4020

22. How many unique arrangements are there of the letters in the word UNUSUAL?

- (a) 120 (b) 720 (c) 840 (d) 1260 (e) 1680

23. The two pictured circles have the same center, but different radii. The chord within the outer circle lies tangent to the inner circle. If the chord has length 6, what is the area of the shaded region?

- (a) 3π (b) 12π (c) 6π
(d) 18π (e) 9π



24. The number 761 satisfies the property that the digits fall in strictly decreasing order. How many three-digit numbers satisfy this property?

- (a) 120 (b) 136 (c) 148 (d) 156 (e) 172

25. Which of the following triplets could be the side lengths of an obtuse scalene triangle?

- (a) 8, 9, 12 (b) 7, 24, 25 (c) 19, 38, 57
(d) 15, 16, 17 (e) 10, 50, 51

26. If A and B are positive integers, where $A < B$, which fraction below is the largest?

- (a) $\frac{A-1}{B-1}$ (b) $\frac{A^2-1}{B^2-1}$ (c) $\frac{A^3-1}{B^3-1}$ (d) $\frac{A+1}{B+1}$ (e) It depends on A & B

27. A line contains the points $P = (3, 5)$ and $Q = (19, 45)$. How many points on this line lie strictly between P and Q and have two integer coordinates?

- (a) 5 (b) 6 (c) 7 (d) 8 (e) 9

28. What is the area of the graph enclosed by $|5x| + |6y| = 30$?

- (a) 7.5 (b) 15 (c) 30 (d) 36 (e) 60

29. An equilateral triangle has the same perimeter as a regular hexagon. What is the ratio of the area of the triangle to the area of the hexagon?

- (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{\sqrt{6}}{3}$ (d) $\frac{\sqrt{3}}{2}$ (e) $\frac{4}{9}$

30. Each problem on the 2010 Utah State Math Contest awards 5 points for a correct answer, 1 point for an omitted answer, and 0 points for a wrong answer. How many integer scores between 0 and 200 are unobtainable?

- (a) 3 (b) 6 (c) 10 (d) 15 (e) 21

31. If $A = 3B$, $5B = 3C$, and $C = 2D$, then the ratio of $\frac{A}{4D}$ is equal to:

- (a) $\frac{3}{4}$ (b) $\frac{8}{15}$ (c) $\frac{9}{10}$ (d) $\frac{5}{12}$ (e) $\frac{72}{5}$

32. On a standard six-sided die, the six faces bear the numbers 1 to 6, and each number has an equal probability of being rolled. You roll the die until the sum of the numbers on your rolls exceeds 12. What is your most likely total?

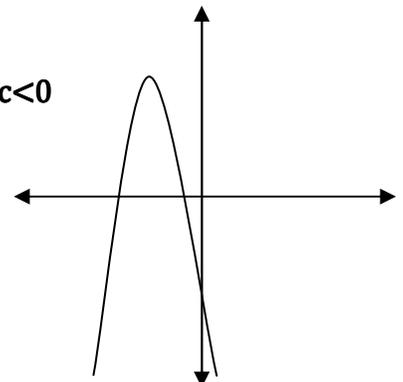
- (a) 13 (b) 14 (c) 15 (d) 16 (e) 13-18 are equally likely

33. If $2^{x+3} + 2^x = 3^{y+2} - 3^y$, and if x and y are integers, then $x + y$ is equal to:

- (a) 5 (b) 6 (c) 7 (d) 8 (e) 9

34. If the graph of the equation $y = ax^2 + bx + c$ is shown, which of the following inequalities is false?

- (a) $a < 0$ (b) $b < 0$ (c) $c < 0$
(d) $\frac{-b}{2a} < 0$ (e) $b^2 - 4ac < 0$



35. Determine the constant term in the binomial expansion of the expression $\left(x^3 + \frac{3}{x^2}\right)^5$:

- (a) 30 (b) 90 (c) 180 (d) 270 (e) 540

36. What is the 15th term in an arithmetic sequence in which the 3rd term is 20 and the 30th term is 2?

- (a) 11 (b) 12 (c) 13 (d) 14 (e) 15

37. A pentagon has exterior angles 52° , $(2x)^\circ$, 25° , $(3x)^\circ$, and 38° . What is x ?

- (a) 13 (b) 31 (c) 49 (d) 67 (e) 85

38. Suppose that the price of game tickets has gone up 4% every year for the past 25 years. If the price of a ticket this year is P , what was the price of a ticket 10 years ago?

- (a) $0.6P$ (b) $\frac{P}{(1.04)^{10}}$ (c) $P(0.6)^{10}$ (d) $\frac{P}{1.4}$ (e) $P(0.96)^{10}$

39. Copy editors look over documents for mistakes. Suppose that a document is checked over by two copy editors; the first editor finds 56 mistakes, and the second finds 50. Upon comparing their results, they find that only 40 mistakes have been found by both of them. For a given editor, each mistake has an equal likelihood of being noticed, though each editor has a different likelihood of catching any given mistake. Assuming that the two editors worked independently, what is the most likely count of the mistakes that both editors have missed?

- (a) 4 (b) 9 (c) 12 (d) 14 (e) 20

40. The included picture contains seven concentric circles which have integer radii numbering from 1 to 7. How many non-congruent pairs of enclosed regions having equal area can be found in this picture?

- (a) 0 (b) 1 (c) 2
(d) 3 (e) 4

