Understanding $S^2$ and $\mathbb{H}^2$ Using Euclidean Space

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GSAC Colloquium

March 28, 2017
There are only 15 known convex pentagons that give monohedral tessellations of the plane.
Bird Fish (M.C. Escher, 1938)

You can see the underlying rhombuses used to create this print.
Lizard 2 (M.C. Escher)
Jos Leys uses Ultrafractal to tessellate the Poincaré disc using a tessellation of the plane. Here he uses Escher’s Lizard print.
Escher started exploring how to fit a repeated pattern in a confined space by having the pattern shrink as it approached a point.
Escher was shocked when he saw this as it answered his question of how to have a pattern approach infinite smallness.
These are D. Dunham’s thoughts on how Escher may have constructed Circle Limit I.
Circle Limit III (M.C. Escher, 1959)
These are Dunham’s thoughts on how Escher may have constructed Circle Limit III.
Circle Limit IV (M.C. Escher, 1960)
Angel-Devil (M.C. Escher, 1941)

This is a flat version of a similar tiling of Circle Limit IV made several years earlier.
Sphere Surface with Fish (M.C. Escher, 1958)
References

- Kathryn Mann’s notes *DIY hyperbolic geometry* for Mathcamp 2015
- *A Tale Both Shocking and Hyperbolic* by Douglas Dunham (appearing in the April 2003 edition of Math Horizons)
- https://www.wikiart.org/en/m-c-escher/all-works