There are five problems on this exam. You may attempt as many problems as you wish; two correct solutions count as a \textit{pass}, and three correct as a \textit{high pass}. Show all your work, and provide reasonable justification for your answers.

1. Let \( d \) be the greatest common divisor of positive integers \( n \) and \( m \). Prove that
   \[
   \mathbb{Z}/(n) \otimes \mathbb{Z}/(m) \cong \mathbb{Z}/(d).
   \]

2. Let \( M \) be the cokernel of the map
   \[
   \begin{pmatrix}
   2 & 2 & 4 \\
   2 & 4 & 6 \\
   2 & 6 & 8 \\
   \end{pmatrix}
   \begin{pmatrix}
   \mathbb{Z}^3 \\
   \mathbb{Z}^3
   \end{pmatrix}
   \]
   Write \( M \) as a direct sum of cyclic \( \mathbb{Z} \)-modules.

3. Let \( I_k \) denote the \( k \times k \) identity matrix. Suppose \( M \) is a \( 2n \times 2n \) matrix over \( \mathbb{Q} \) such that \( M^2 = -I_{2n} \), prove that \( M \) is similar to the matrix
   \[
   \begin{pmatrix}
   0 & -I_n \\
   I_n & 0 \\
   \end{pmatrix}
   \]

4. State the Noether normalization lemma.
   Let \( M \) be a maximal ideal of the ring \( A = \mathbb{R}[x,y,z]/(x^2 + y^2 + z^2 + 1) \). Determine the field \( A/M \).

5. Consider the ring homomorphism given by
   \[
   \begin{align*}
   \mathbb{Z}[x] & \longrightarrow \mathbb{Z} \times \mathbb{Z} \\
   f(x) & \longmapsto (f(1), f(-1))
   \end{align*}
   \]
   Is this homomorphism surjective? Determine a minimal generating set for the kernel.