UNIVERSITY OF UTAH DEPARTMENT OF MATHEMATICS

Ph.D. Preliminary Examination in Applied Mathematics

Applied Complex Variables and Asymptotic Methods

August 20, 2020

Instructions:	This examination includes five problems, but you are to work three of them. If you work more than three, then state which problems you wish to be graded, otherwise the first three will be graded. All problems are worth 10 points; *Pass* - 20; *High pass* - 25.

1. What are the values of the following complex expressions? Show them in the complex plane.
   (a) i^i
   (b) $\tan i$

2. The following functions $f(x)$ are expanded in powers of x (i.e. in Taylor or Laurent series around the origin); x is a real variable, $f(x)$ is a real-valued function. Find the intervals of convergence of the series.
   (a) $f(x) = \frac{1}{\sin x}$
   (b) $f(x) = \frac{1}{\sin x \ + \ 2}$

3. (a) Formulate and prove the Argument Principle.
   (b) Show that it implies the Fundamental Theorem of Algebra.

4. Calculate the following integrals
   (a) $\int_0^\infty \frac{\sin x}{x} \, dx$
   (b) $\int_0^\infty \frac{x^{\sqrt{2}}}{x+1} \, dx$

5. Suppose domain D (of the x, y-plane) is mapped onto domain Δ (of the ξ, η-plane) by an analytic function f: $\xi + i\eta = f(x + iy)$, and $f'(x + iy) \neq 0$ at any point $(x, y) \in D$.
   (a) Explain why this map preserves "small" shapes (and so, the map can be called *conformal*).
   (b) Explain why this map preserves the form of the Laplace equation.