University of Utah, Department of Mathematics
Algebra 2 Qualifying Exam
August 2020

There are five problems on this exam. You may attempt as many problems as you wish; two correct solutions count as a *pass*, and three correct as a *high pass*. Show all your work, and provide reasonable justification for your answers.

1. (a) Find a monic irreducible degree two polynomial $f(y) \in \mathbb{F}_3[y]$ that is not equal to y^2+1.
   (b) For the $f(y)$ you found above, construct an explicit isomorphism of fields $\mathbb{F}_3[x]/(x^2+1) \xrightarrow{\sim} \mathbb{F}_3[y]/(f(y))$.

2. Suppose $K/\mathbb{Q}$ is a Galois extension, and L/K is a Galois extension. Is $L/\mathbb{Q}$ a Galois extension? If yes, prove it, and otherwise give a counterexample.

3. Show that $\mathbb{Q}(\cos(2\pi/7))/\mathbb{Q}$ is Galois, and compute its Galois group.

4. Show there is no simple group of order $107421875 = 5^{10} \cdot 11$.

5. Find an N such that every abelian group with at most 5 elements is a quotient of $(\mathbb{Z}/N\mathbb{Z})^\times$.