1. Let $G$ be a group of order $p^3$, where $p$ is a prime. Suppose $G$ is not abelian. Show that the center $Z$ of $G$ is isomorphic to $\mathbb{Z}/(p)$ and that $G/Z \cong \mathbb{Z}/(p) \times \mathbb{Z}/(p)$.

2. What is the number of elements of order 11 in a simple group of order 660?

3. Let $M$ be the cokernel of the map $\mathbb{Z}^2 \rightarrow \mathbb{Z}^3$ given by
   \[
   \begin{pmatrix}
   3 & 6 \\
   4 & 10 \\
   10 & 22 
   \end{pmatrix}.
   \]
   Write $M$ as a direct sum of cyclic groups.

4. Let $M$ be an $n \times n$ matrix with entries from a field $\mathbb{F}$ such that $M^3 = I$. Is $M$ necessarily diagonalizable if (a) $\mathbb{F} = \mathbb{Q}(\sqrt{3})$, (b) $\mathbb{F} = \mathbb{Q}(i)$, (c) $\mathbb{F} = \mathbb{F}_3$?

5. Determine all $3 \times 3$ matrices $M$ with entries from $\mathbb{Q}$ such that $M^8 = I$ and $M^4 \neq I$.

6. Find all positive integers $n$ such that $\cos(2\pi/n)$ is rational.

7. Is the polynomial $x^8 + x + 1$ irreducible in $\mathbb{F}_2[x]$?

8. Which of the following is a principal ideal domain? (a) $\mathbb{Z}[i]$, (b) $\mathbb{Z}[2\sqrt{2}]$, (c) $\mathbb{Q}[3\sqrt{3}]$.

9. Let $\mathbb{F}$ be a field with $\mathbb{Q} \subset \mathbb{F} \subset \mathbb{C}$ such that $[\mathbb{F} : \mathbb{Q}]$ is odd.
   (a) If $\mathbb{F}/\mathbb{Q}$ is Galois, prove that $\mathbb{F}$ is contained in $\mathbb{R}$.
   (b) Find an extension with $[\mathbb{F} : \mathbb{Q}] = 3$ such that $\mathbb{F}$ is not contained in $\mathbb{R}$.

10. Prove that each element of a finite field can be written as a sum of two squares.