(1) Show that a finite group of order 105 has a non-trivial normal subgroup.
(2) Let $p$ be a prime. Determine all groups of order $p^2$.
(3) Let $R$ be a commutative ring with 1. Assume that $R$ satisfies the ascending chain condition. Let $I$ be an ideal generated by an infinite sequence of elements $x_1, x_2, \ldots$ in $R$. Show that $I$ is finitely generated.
(4) Let $\omega = -\frac{1}{2} + \frac{\sqrt{-3}}{2}$, a cube root of 1. Show that the ring $\mathbb{Z}[\sqrt{-3}]$ is a euclidean domain.
(5) Let $A$ be a rational $3 \times 3$ matrix such that $A^3 = A$. Show that $A$ can be diagonalized.
(6) Describe all finitely generated $\mathbb{Z}$-submodules of $\mathbb{Q}$.
(7) Show that $1 \otimes (1, 1, \ldots) \neq 0$ in the tensor product $\mathbb{Q} \otimes_{\mathbb{Z}} \prod_{n=2}^{\infty} (\mathbb{Z}/n\mathbb{Z})$.
(8) Prove that $\Phi_{2^n}(x) = x^{2^{n-1}} + 1$ is irreducible in $\mathbb{Q}[x]$.
(9) Find a real number $\alpha$ such that $\mathbb{Q}(\alpha)$ is a Galois extension of $\mathbb{Q}$ with the Galois group $\mathbb{Z}/5\mathbb{Z}$.
(10) Let $G$ be the group of symmetries of an equilateral triangle. Show that the 2 dimensional representation (as symmetries of the triangle) is irreducible by calculating the character table.