1. Determine the number of 5-Sylow subgroups of $\text{SL}_2(\mathbb{F}_5)$.

2. Let $G$ be the subgroup of $\text{GL}_2(\mathbb{R})$ consisting of matrices of the form \[
\begin{pmatrix}
a & c \\
0 & b
\end{pmatrix}.
\] Is $G$ solvable?

3. Consider the automorphisms $\sigma, \tau$ of $\mathbb{Q}(x)$ with $\sigma: x \mapsto 1/x$ and $\tau: x \mapsto 1 - x$. What is the order of the group generated by these two elements? Determine the group.

4. Set $R = \mathbb{Q}[x]$, and consider the submodule $M$ of $R^2$ generated by the elements $(1 - 2x, -x^2)$ and $(1 - x, x - x^2)$. Express $R^2/M$ as a direct sum of cyclic modules.

5. Recall that a Hermitian matrix is a complex matrix which equals its conjugate transpose. Determine the conjugacy classes of $5 \times 5$ Hermitian matrices $A$ satisfying $A^5 + 2A^3 + 3A = 6I$.

6. Determine the number of conjugacy classes of $4 \times 4$ complex matrices satisfying $A^3 - 2A^2 + A = 0$.

7. Let $\alpha$ be the positive real root of $x^6 - 7$. What is the number of elements of the ring $\mathbb{Z}[\alpha]/(\alpha^2)$? Is every ideal in this ring principal?

8. Find the degree of the splitting field of $x^6 - 3$ over (i) $\mathbb{Q}(\sqrt{-3})$ and (ii) $\mathbb{F}_5$.

9. Prove that $x^4 + 1$ is reducible over any field of positive characteristic.

10. For $p$ a prime, determine the Galois group of $x^p - 2$ over $\mathbb{Q}$. What is its order? Is it abelian?