Abstract

Exact relations between critical exponents for elastic stiffness and electrical conductivity of percolating networks

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Received: Thursday, April 18,2002

It has long been known that the critical exponent T of the elastic stiffness $C_e \propto \Delta p^T$ of a d-dimensional percolating network ($\Delta p \equiv p - p_c > 0$ measures the closeness of the network to its percolation threshold p_c) satisfies the following inequalities $1 + d\nu \leq T \leq t + 2\nu$, where t is the critical exponent of the electrical conductivity $\sigma_e \propto \Delta p^t$ of the same network and ν is the critical exponent of the percolation correlation length $\xi \propto \Delta p^{-\nu}$. Similarly, the critical exponents which characterize the divergences $C_e \propto |\Delta p|^{-S}$, $\sigma_e \propto |\Delta p|^{-s}$ of a percolating rigid/normal network (i.e., a random mixture of normal elastic bonds and totally rigid bonds) and a percolating superconducting/normal network (i.e., a random mixture of normal conducting bonds and perfectly conducting bonds; $\Delta p \equiv p - p_c < 0$ now measures the closeness of the rigid or superconducting constituent to its percolation threshold p_c) have long been known to satisfy $S \leq s$. I now show that, when $d = 2$ or $d = 3$, T is in fact exactly equal to $t + 2\nu$ and S is exactly equal to s. This is achieved by a judicious use of some variational principles for electrical and elastic networks, and by a judicious treatment of constraints and short range correlations in those networks. An extension of these proofs to arbitrary (integer) values of the dimensionality d should be possible.