Abstract

Nonlinear Interactions of Wavepackets in a Periodic Media

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We develop a consistent mathematical theory that describes nonlinear interaction of wavepackets in a nonlinear periodic dielectric media for the dimensions one, two and three. The theory is based on the Maxwell equations with quadratic and cubic constitutive relations. Solutions of the Maxwell equations on long time intervals are expanded in convergent series with respect to a small parameter alpha, which measures the contribution of the nonlinearity. After that we investigate in detail the principal term of the expansion in the case where the excitations (and, consecutively, solutions) have a form of wavepackets. The ratio of the amplitude frequency and the carrier frequency of a wavepacket is an important small parameter rho. The principal term describes the nonlinear interaction of a continuum of modes and is written in the form of an oscillatory integral. The phase function of the integral is written in terms of the Floquet-Bloch dispersion relations of the periodic media. We consider the situation when the stationary phase method is applicable, that means that initially the spatial extension of the wavepacket is larger then the period and much smaller then the domain where the medium is periodic. A detailed mathematical analysis shows that the interaction integral expands into sum of terms with different powers of rho and the leading terms correspond to only a few interacting modes. We give a classification of the nonlinear interactions between wavepackets in a media with generic dispersion relations based on the powers. The powers take on only a relatively small number of prescribed values collected in a table, their values depend on a type of degeneracy of phase functions formed by the dispersion relations of the media. The crucial role in selecting the strongest interactions, in particular the second harmonic generation in a quadratic medium, is played by internal symmetries of the phase function.

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