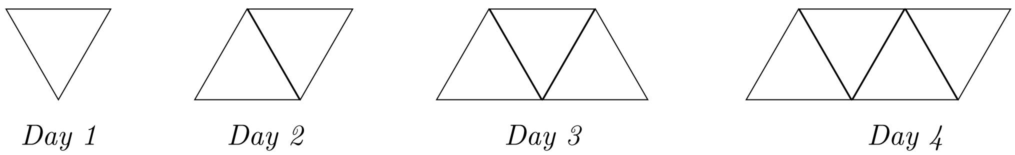
8 Handouts

8.1 Visual Patterns

Exercise 1 *Look at the pattern below and answer the questions:*



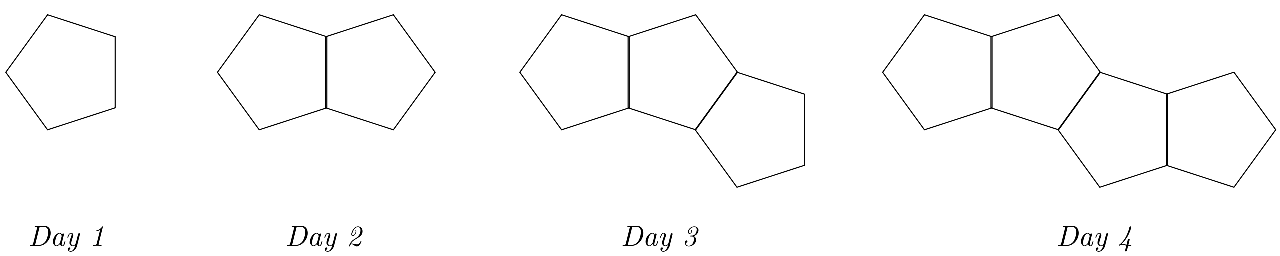
a. Describe the pattern that you see in the sequence of figures above.

b. Draw the figure that would appear on the fifth day.

c. The figure on the second day requires 5 line segments. How many line segments are needed on the fifth day?

d. How many line segments are needed on the 25^{th} day?

Exercise 2 *Look at the pattern below and answer the questions:*



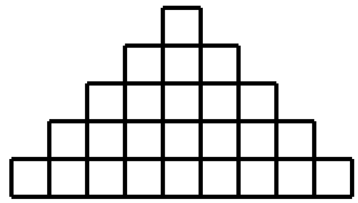
a. Describe the pattern that you see in the sequence of figures above.

b. Assuming the pattern continues in the same way, draw the figure that occurs on the fifth day.

c. The side length of each pentagon is 1. What is the perimeter of the figure on fifth day?

d. What is the perimeter of the figure on day 30?

Question 8.1 Look at the figure below and answer the questions that follow.



a. How many squares are in the top row?

b. How many squares are in the second row?

c. How many squares are in the fourth row?

d. If the figure were extended indefinitely forever, how many squares would be in the n^{th} row?

e. How many unit squares are in the first row? (a unit square is the smallest one in the picture)

f. How many unit squares are in the first two rows?

g. How many unit squares are in the first n rows?

h. What is the sum of the first n odd numbers?

8.2 Tiling a Pool

Exercise 3 *A cafeteria in a school has square tables where students can eat lunch in groups of four. If six students want to eat lunch at the same table, then they can push two tables together to accommodate their group; even larger groups can be handled by joining together more tables in a straight line.*

a. Draw diagrams representing this sequence.

b. Construct a sequence that models this situation and fill in a table for the first several members of the sequence.

c. If possible, describe both recursive and explicit rules for the sequence.

d. Draw a graph representing this situation.

e. Find the value of the 32^{nd} member of the sequence. What does the 32^{nd} term in the sequence tell us?

f. Is there a member of the sequence whose value is 324? How do you know?

Exercise 4 *For each arithmetic sequence below, find the common difference, and write the nth term in terms of n:*

a. $2, 7, 12, 17, 22, \ldots$

b. $2+1\cdot5, 2+2\cdot5, 2+3\cdot5, \ldots$

c. $y+1\cdot5, y+2\cdot5, y+3\cdot5, \ldots$

d. $y+1\cdot x, y+2\cdot x, y+3\cdot x, \ldots$

Exercise 5 *The following sequences are arithmetic. Find the missing terms.*

a. $7, \square, 15, \ldots$

b. $10, \square, \square, -14, \ldots$

c. $15, \square, \square, \square, \square, 15, \ldots$

Exercise 6 *Here is a picture of a candy machine at the Gateway Mall (400 W 100 S). Each time a costumer inserts a quarter, 13 candies come out of the machine. The machine holds 14 pounds of candy. Each pound of m&m's contains 220 individual candies.*

a. *How many candies are in the machine when it is full?*

b. *How many candies are in the machine after 1 customer? How many are in the machine after 2 customers?*

c. *The amount of candies in the machine after n customers can be modeled using an arithmetic sequence c_n. Write down the explicit formula for c_n. What is the relationship between c_n and c_{n+1}?*

d. *When does our model stop making real world sense?*

e. *To avoid theft, the owners of the machine don't want to let too much money collect in the machine, so they take all the money out when they think the machine has about $30 in it. The tricky part is that the store owners can't tell how much money is actually in the machine without opening it up, so they choose when to remove the money by judging how many candies are left in the machine. About how full should the machine look when they take the money out? How do you know?*

Exercise 7 *The following sequence is an arithmetic sequence:* $2, 5, 8, \ldots$

a. What is the common difference?

b. What is the 12-th term? What is the n-th term?

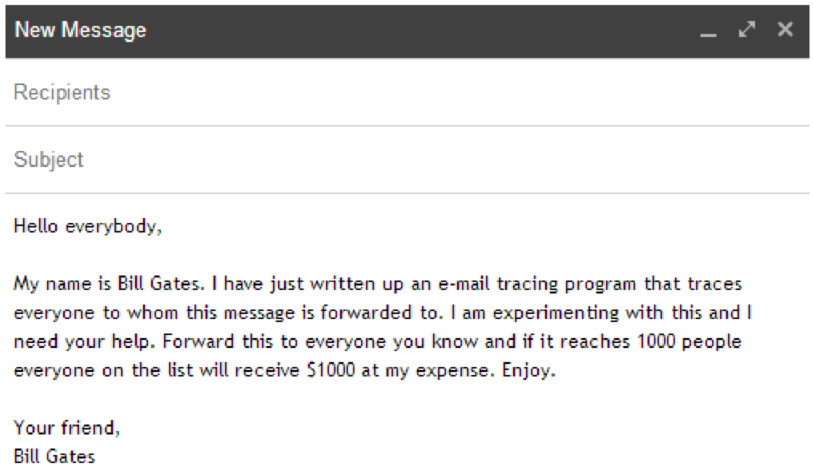
c. Graph the first 5 terms.

Exercise 8 *The sum of the first four terms of an arithmetic sequence is 10. If the fifth term is 5, what is the sixth term?*

Exercise 9 *The second and ninth terms of an arithmetic sequence are 2 and 30, respectively. What is the fiftieth term?*

8.3 Geometric Sequences

Exercise 10 *The following message began circulating on the Internet around 21 November 1997:*



New Message

Recipients

Subject

Hello everybody,

My name is Bill Gates. I have just written up an e-mail tracing program that traces everyone to whom this message is forwarded to. I am experimenting with this and I need your help. Forward this to everyone you know and if it reaches 1000 people everyone on the list will receive $1000 at my expense. Enjoy.

Your friend,
Bill Gates

The first step in the chain has Bill Gates sending the email out to his 8 closest friends. Assume that everyone that receives the email forwards it to 10 people. So the second step in the chain has 8 friends each sending out 10 emails, therefore a total of 80 people receive the email in the second step.

a. How many people receive the email in the third step?

b. How many people receive the email in the fourth step?

c. Let's model how many people receive the email on the n-th day using a geometric sequence e_n. Write an explicit formula for e_n.

d. How is e_n related to e_{n-1}?

e. After the second step a total of 88 people received the email. How many total people received the email after the third step?

f. How many steps does it take the chain letter to reach 1000 recipients?

g. If after the fifth step Bill Gates realizes it is getting out of control and puts an end to the chain letter, how much money is he on the hook for?

Exercise 11 *The following sequence is a geometric sequence:* $2, 10, 50, 250, \ldots$

a. What is the common ratio?

b. What is the n-th term?

c. What is the recursive definition for this sequence?

d. Graph the first 5 terms.

Exercise 12 *For each geometric sequence below, find the common ratio, write the formula for n-th term, and find 12th term of the sequence:*

a. $2, 6, 18, 54, \ldots$

b. $2 \cdot 3, 2 \cdot 9, 2 \cdot 27, \ldots$

c. $25, 5, 1, \frac{1}{5}, \ldots$

d. $y \cdot x, y \cdot x^2, y \cdot x^3, \ldots$

Exercise 13 *A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?*

Exercise 14 *Each sequence below is either arithmetic or geometric. First decide if the sequence is arithmetic or if it is geometric, find the next two terms of the sequence, find 83 term of the sequence, then give the explicit formula for each sequence:*

a. $54, 18, 6, \ldots$

b. $2 \cdot 3, 2 \cdot 3 + 4, 2 \cdot 3 + 8, \ldots$

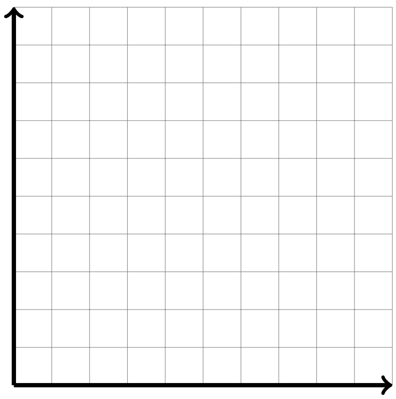
c. $-3, -4, -5, -6, -7, -8, \ldots$

d. $25, -5, 1, -\frac{1}{5}, \ldots$

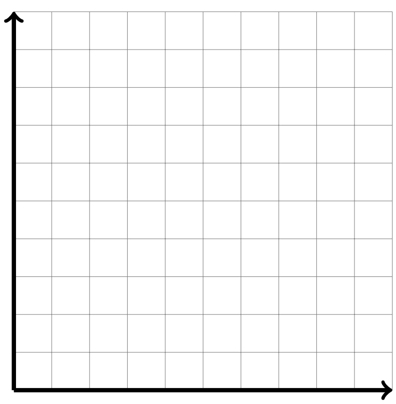
e. $ab, a^2b^3, a^3b^5, \ldots$

Exercise 15 *The following sequences are neither geometric or arithmetic. Graph the terms given for each sequence and describe how the graph shows that the sequences are not arithmetic.*

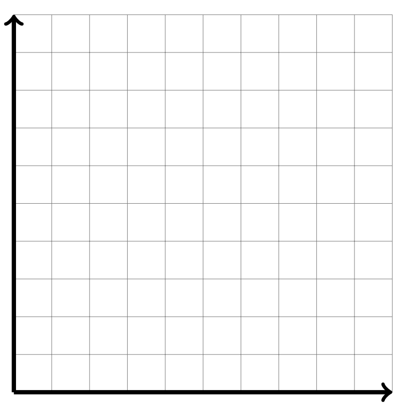
a. $5, 8, 13, 20, 29, 40, 53, 68, \ldots$



b. $7, 13, 23, 37, 55, \ldots$



c. $-2, 7, 22, 43, 70, \ldots$



Exercise 16 *The terms* $140, a, \frac{45}{28}$ *are the first, second and third terms, respectively, of a geometric sequence. If* a *is positive, what is the value of* a*?*

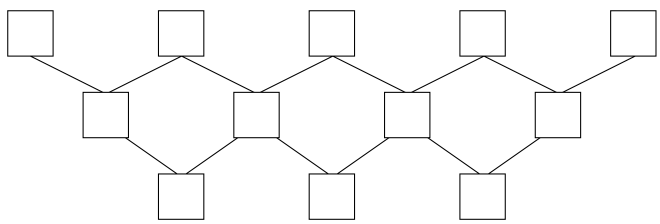
8.4 Counting High-Fives

Exercise 17 *Let* $\{t_1, t_2, \ldots\}$ *be a sequence where the* n*-th term is given by* $4n^2 - n + 4$.

1. Fill in the following table:

n	t_n
1	
2	
3	
4	
5	
6	
7	

2. Calculate a few second differences of this sequence. To help you organize your thoughts, use the following diagram:

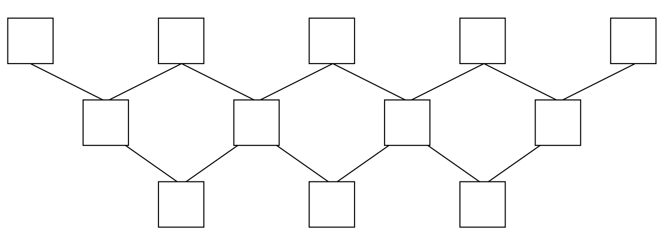


Exercise 18 *Let* $\{q_1, q_2, \ldots\}$ *be a sequence where the* n*-th term is given by* $3n^2 - n + 1$.

1. Fill in the following table:

n	q_n
1	
2	
3	
4	
5	
6	
7	

2. Calculate a few second differences of the hand shake sequence. To help you organize your thoughts, use the following diagram:



Exercise 19 *Let S_n be the sum of the first n positive integers, for example* $S_5 = 1+2+3+4+5$.

a. Fill in the following table:

b. Is this sequence similar to another sequence we have studied? Use this information to write down an explicit formula for S_n.

n	S_n
1	
2	
3	
4	
5	
6	
7	

The following captures a beautiful argument attributed to Gauß to compute S_{100}.

$$S_{100} = 1+2+3+\ldots+98+99+100$$

$$= (1+100)+(2+99)+(3+98)+\ldots+(50+51)$$

$$= \underbrace{101+101+101+\ldots+101}_{\text{There are fifty 101's}} = 50\cdot 101 = 5050$$

c. Which algebraic rules are being used when we write the second equality?

d. Use this argument to verify our formula for S_n.

Exercise 20 *Let $s_1, s_2, s_3, \ldots$ be the sequence such that s_n is the sum of the first n even numbers, for example:* $s_5 = 2+4+6+8+10 = 30$.

a Fill in the following table

n	1	2	3	4	7	n
s_n	2					

b Is s_n a geometric or arithmetic sequence?

c What is the relationship between s_n and s_{n-1}?

d Write down the algebraic expression for s_n.

8.5 More Sequences

Exercise 21 *Each spring, a fishing pond is restocked with fish. That is, the population decreases each year due to natural causes, but at the end of each year, more fish are added. Here's what you need to know.*

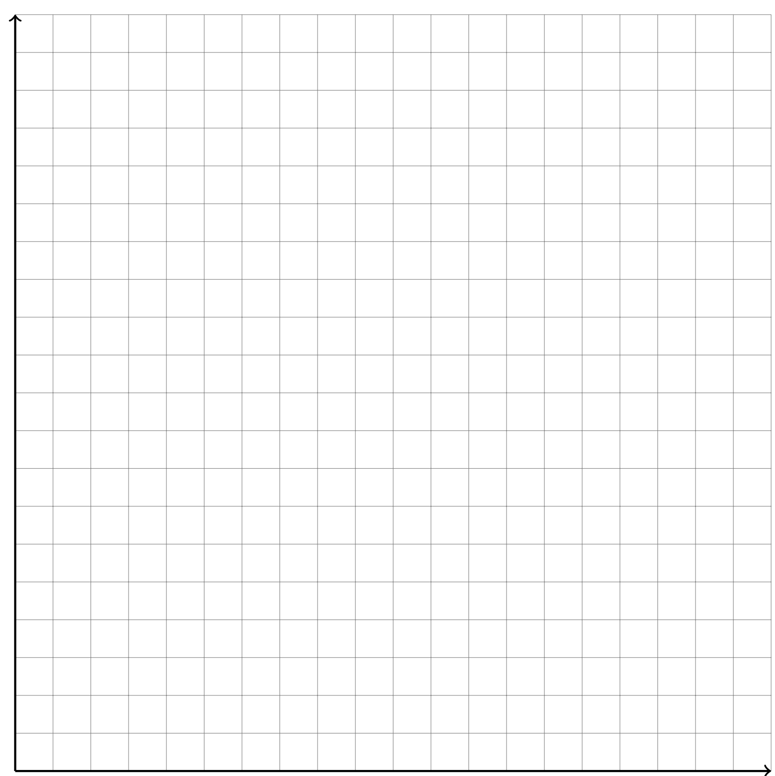
- *There are currently* 3000 *fish in the pond.*
- *Due to fishing, natural death, and other causes, the population decreases by* 20% *each year.*
- *At the end of each year,* 1000 *new fish are added to the pond.*

Using the information provided we will try to understand what happens to this population of fish in the long term.

a. Fill in the table that will contain the amount of fish in the pond during the first 15 years.

Year	Number of fish in pond
1	3400
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

b. Graph this data.



c. Explain what seems to be happening to the fish.

d. Let f_n be the number of fish in the pond after n years. What is the relationship between f_n and f_{n-1}?

e. Predict approximately how many fish will be in the pond after 40 years.

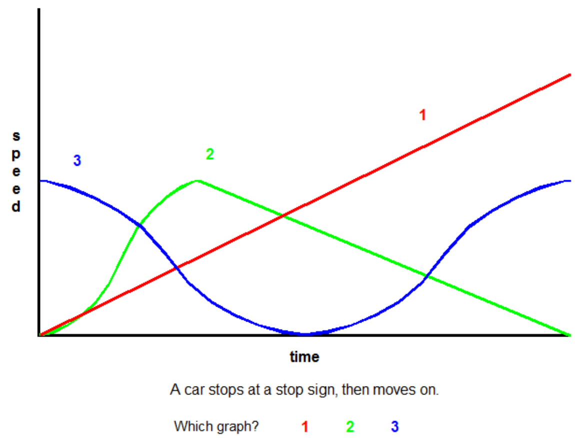
Exercise 22 *Suppose you put 2 cents in a jar today and each day thereafter you triple the amount you put in the previous day. How much would you put in on the 17th day? How big must your jar be?*

Exercise 23 *Is there a sequence that can be claimed to be both arithmetic and geometric?*

Exercise 24 *Miguel was asked to consider the pattern* $0, 5, 10, 15, ...$ *and list the next term. Miguel said 24. Can you figure out why Miguel chose that instead of 20?*

8.6 Describing Relationships

Exercise 25 *Imagine you are driving to school. You come to a stop sign and stop, then move on. Which of the graphs depicted below best represents the speed of your car?*



a. Write couple of sentences explaining why the graph you chose describes the situation above.

b. Write couple of sentences explaining what situation you would observe if the car's speed had been described by the other graphs.

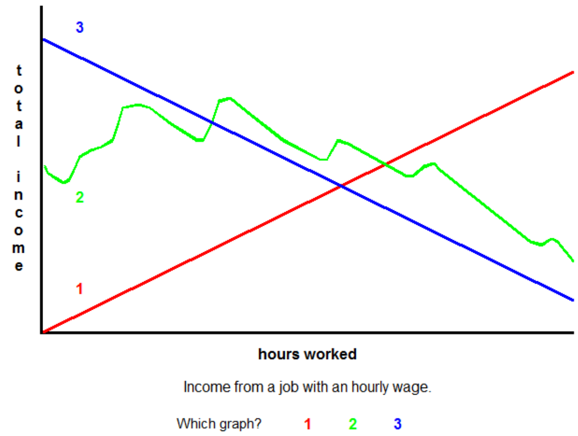
c. Put a scale on your graph. On the horizontal axis place 0-60 second, and on the vertical axis place 0-35 miles per hour.

- How fast was the car moving after 30 seconds? 45 ? 60?

- At what time was the car going 20 miles per hour? How many seconds had passed when you were going 0 miles per hour?

- Complete the following pairs so each belongs to the graph: (,0), (10,), (25,), (,25).

Exercise 26 *If you work an hourly wage job, which graph would best depict your total wage?*



a. Write couple of sentences explaining why the graph you chose describes the situation above.

b. Write couple of sentences explaining what your wages would be like if the other two graphs were the correct graphs.

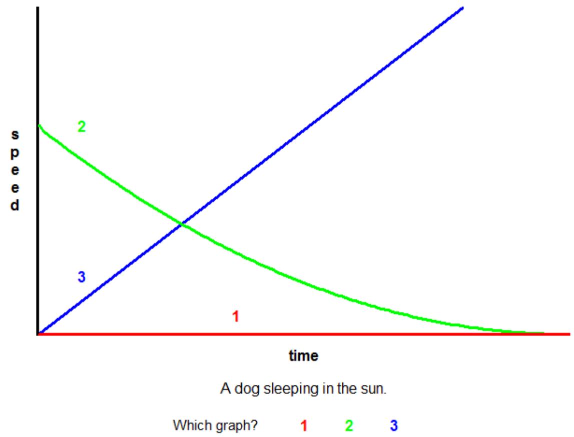
c. Put a scale on your graph. On the horizontal axis place 0-8 hours, and on the vertical axis place 0-100 dollars.

- How much money did you make after 2 hours? After 3 hours?

- For you to make 50 dollars how long did you have to work?

- How much do you make per hour?

Exercise 27 *Your dog spends a full day sleeping in the sun (oh, to be a dog!). Which graph best depicts your dog's movement?*



a. Write couple of sentences explaining why the graph you chose describes the situation above.

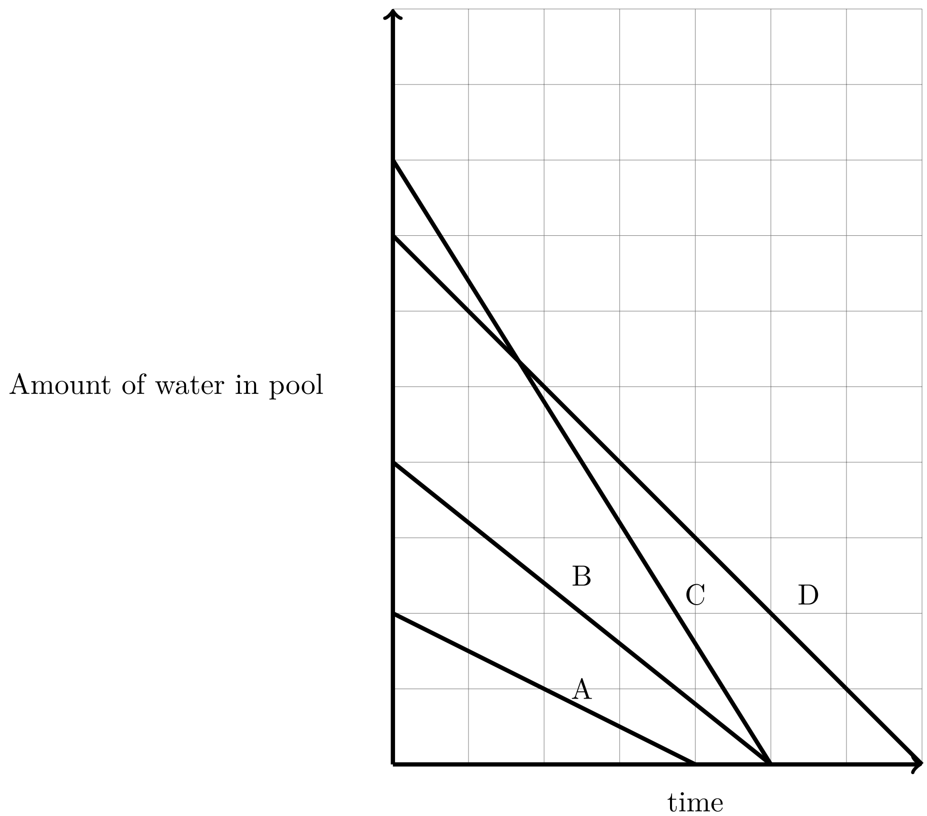
b. Write couple of sentences explaining your dog's activities if the other two graphs were correct graphs.

c. Put a scale on your graph. On the horizontal axis place 0-12 hours, and on the vertical axis place 0-10 miles per hour.

- How fast was your dog moving after 3 hours? After 6 hours?

- What was the time when your dog was moving 3 miles per hour? 0 miles per hour?

Exercise 28 *Each graph below represents the amount of water in pool as it is being drained by a pump.*



Carefully explain the answers to each question using the scale on the graph.

a. Which pool had the most water to begin with?

b. Which pool was empty first?

c. Which pump pumps the most water per minute?

8.7 Relations and Functions

Exercise 29 *Do the following sets of ordered pairs represent functions? Explain.*

a. {(1,0), (0,1), (3, 2), (5,0)}

b. {(-5,4), (4,4), (7.6, 3.87), (9, 213)}

c. {(1,0), (2,3), (1,6), (5,8)}

d. {(happy, purple), (bumble bee, -6.78), (grumpy, potato salad), (&, 0.001dog), (#,)}

Exercise 30 *Do the following tables represent functions? Explain.*

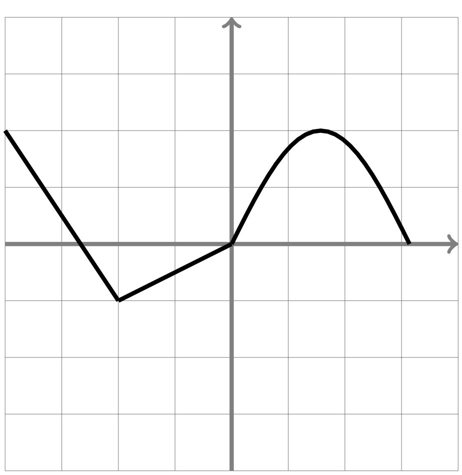
x	$a(x)$
1	-1
2	3
3	7
1	-1
4	11

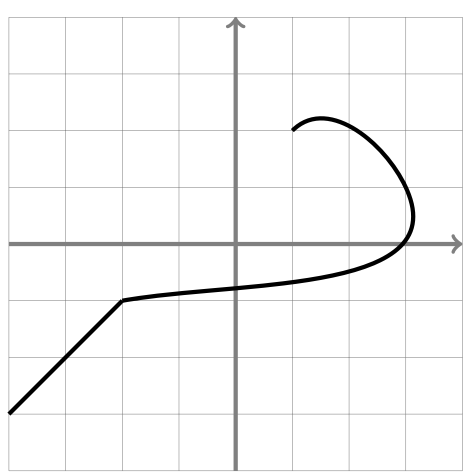
x	$b(x)$
A	A
B	C
C	D
D	E
E	A

x	$c(x)$
1	-1
2	3
4	11
-2	-5
2	2

x	$d(x)$
1	5
2	5
3	5
4	5
12	5

Exercise 31 *Are the following graphs, graphs of some function? Explain.*





Explanation.

8.8 Ways to represent a function

Exercise 32 *Is it always possible to represent a function by an equation? Explain.*

Exercise 33 *We have a function* $h : \mathbb{R} \to \mathbb{R}$ *given by an algebraic rule* $h(x) = -x^2 + 2x - 1$.

a. Complete the following table:

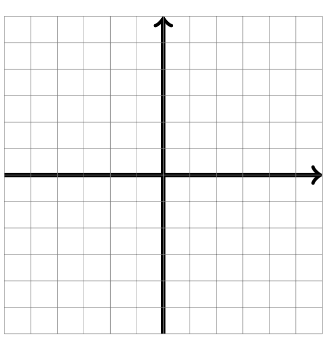
x	$h(x)$
-2	
0	
$\frac{1}{2}$	
2	
2.5	

b. Does the point $(-1, -4)$ lie on the graph of h? How do you know?

c. Does the point $(0, -1)$ lie on the graph of h? How do you know?

d. Find a few more ordered pairs for h.

e. Graph h.



f. Use the graph to approximate the input x such that $h(x) = 0$.

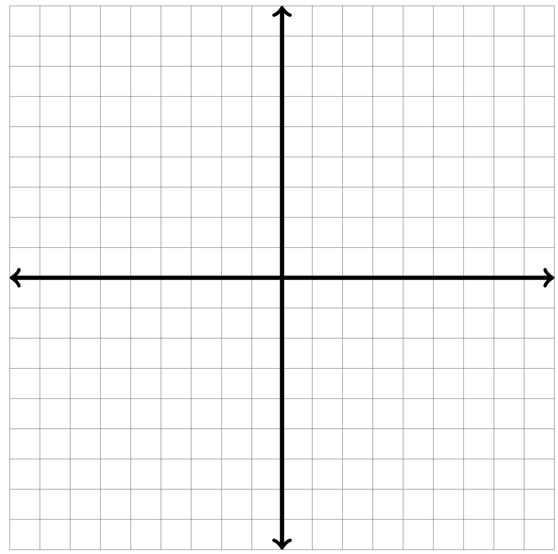
Exercise 34 *A spherical balloon becomes bigger and bigger as it is filled with more and more air, although it always retains its spherical shape. It starts out at* $t = 0$ *as a balloon with a volume of 10 cubic centimeters. With every second, 30 cubic centimeters more of air is pumped into the balloon. Make a table, graph and find the expression for the function that gives volume as a function of time (in other words, the time is the input and the volume is the output).*

Exercise 35 *Let $f : \mathbb{R} \to \mathbb{R}$ be a function given by $f(x) = -3x + 1$.*

a. Construct a table for $f : \mathbb{R} \to \mathbb{R}$.

b. Plot the entries from your table on a graph.

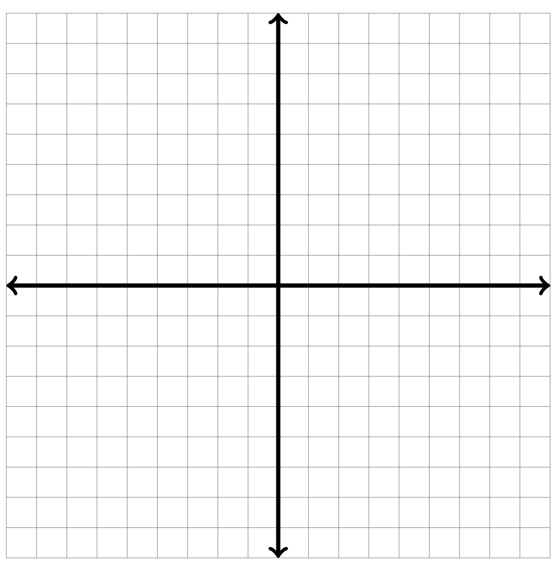
x	$f(x)$



Exercise 36 *Graph the function $f : \mathbb{R} - \{1\} \to \mathbb{R}$ given by the algebraic rule*

$$f(x) = \frac{x-2}{x-1}.$$

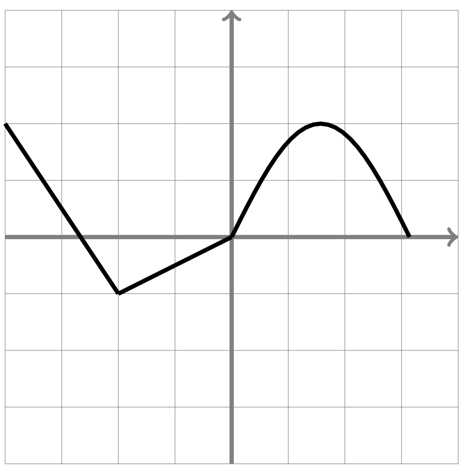
Explain briefly why the domain of f isn't the whole $\mathbb{R}$.



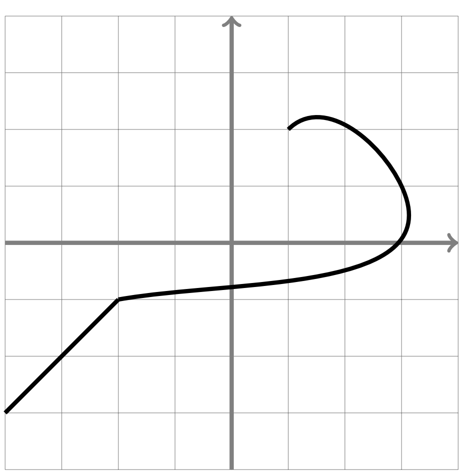
Exercise 37 *An x-intercept is a point where a graph of a function crosses the x-axis. How many x-intercepts does $y = -\frac{1}{2}x + 3$ have? How many x-intercepts does $y = x(x-2)$ have?*

Exercise 38 *A y-intercept is a point where a graph of a function crosses the y-axis. No function can have more than one y-intercept. Explain why not.*

Exercise 39 *For each of the relations below make a table of values which includes every integer input that can be read from the graphs.*



x	$f(x)$



x	$g(x)$

8.9 Combining Functions

Exercise 40 *In each question below f and g are functions defined by an algebraic rule. For each pair of functions find a simplified formula for the required combination of functions, then evaluate those functions at 1 and -1:*

a. $f(x) = x^2 + 1$; $g(x) = x - 3$.

(a) $(f+g)(x) =$

$(f+g)(1) =$ $(f+g)(-1) =$

(b) $(f-g)(x) =$

$(f-g)(1) =$ $(f-g)(-1) =$

(c) $(f \cdot g)(x) =$

$(f \cdot g)(1) =$ $(f \cdot g)(-1) =$

(d) $(f \div g)(x) =$

$(f \div g)(1) =$ $(f \div g)(-1) =$

(e) $(f \circ g)(x) =$

$(f \circ g)(1) =$ $(f \circ g)(-1) =$

(f) $(g \circ f)(x) =$

$(g \circ f)(1) =$ $(g \circ f)(-1) =$

b. $f(x) = 10 - x$; $g(x) = \sqrt{x}$.

(a) $(f+g)(x) =$

$(f+g)(1) =$ $(f+g)(-1) =$

(b) $(f-g)(x) =$

$(f-g)(1) =$ $(f-g)(-1) =$

(c) $(f \cdot g)(x) =$

$(f \cdot g)(1) =$ $(f \cdot g)(-1) =$

(d) $(f \div g)(x) =$

$(f \div g)(1) =$ $(f \div g)(-1) =$

(e) $(f \circ g)(x) =$

$(f \circ g)(1) =$ $(f \circ g)(-1) =$

(f) $(g \circ f)(x) =$

$(g \circ f)(1) =$ $(g \circ f)(-1) =$

c. $f(x) = \sqrt{4x}$; $g(x) = \frac{1}{x}$.

(a) $(f+g)(x) =$

$(f+g)(1) =$ $(f+g)(-1) =$

(b) $(f-g)(x) =$

$(f-g)(1) =$ $(f-g)(-1) =$

(c) $(f\cdot g)(x) =$

$(f\cdot g)(1) =$ $(f\cdot g)(-1) =$

(d) $(f\div g)(x) =$

$(f\div g)(1) =$ $(f\div g)(-1) =$

(e) $(f\circ g)(x) =$

$(f\circ g)(1) =$ $(f\circ g)(-1) =$

(f) $(g\circ f)(x) =$

$(g\circ f)(1) =$ $(g\circ f)(-1) =$

d. $f(x) = 5x+1$; $g(x) = 2x^2-7$.

(a) $(f+g)(x) =$

$(f+g)(1) =$ $(f+g)(-1) =$

(b) $(f-g)(x) =$

$(f-g)(1) =$ $(f-g)(-1) =$

(c) $(f\cdot g)(x) =$

$(f\cdot g)(1) =$ $(f\cdot g)(-1) =$

(d) $(f\div g)(x) =$

$(f\div g)(1) =$ $(f\div g)(-1) =$

(e) $(f\circ g)(x) =$

$(f\circ g)(1) =$ $(f\circ g)(-1) =$

(f) $(g\circ f)(x) =$

$(g\circ f)(1) =$ $(g\circ f)(-1) =$

Exercise 41 *I was at the store and over heard the following conversation:*

Customer: I have a 20% off coupon. I would like you to apply it after the 8% sales tax to maximize my savings.
Cashier: No, you want me to apply it before the tax. That way the tax is applied to a smaller price. Who was correct? Use the language of function composition as you discuss.

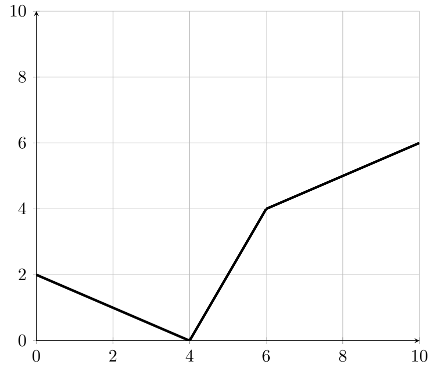
Exercise 42 *Find $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$. Answers may vary.*

a. $h(x) = (3x-5)^4$.

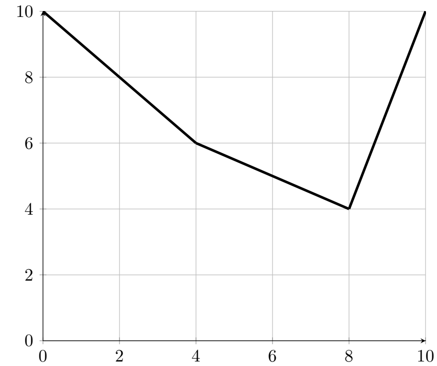
b. $h(x) = \sqrt{9x+1}$.

c. $h(x) = \frac{6}{5x-2}$

Exercise 43 *Two functions are given by their graphs:*



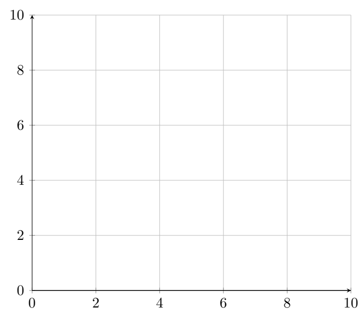
the graph for f



the graph for g

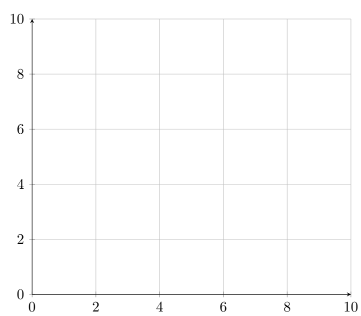
Find a table and graph each of the following functions:

x	$(f+g)(x)$



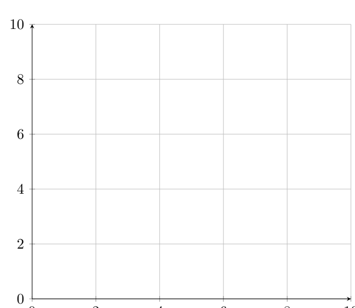
the graph for $(f+g)(x)$

x	$(f-g)(x)$



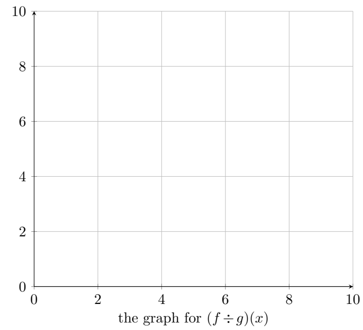
the graph for $(f-g)(x)$

x	$(f\cdot g)(x)$

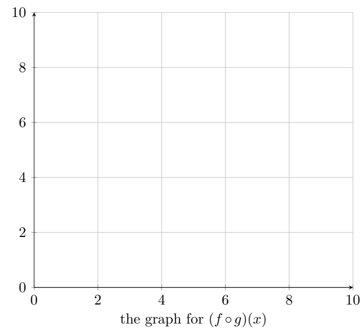


the graph for $(f\cdot g)(x)$

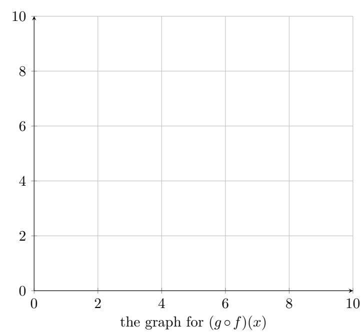
x	$(f \div g)(x)$



x	$(f \circ g)(x)$



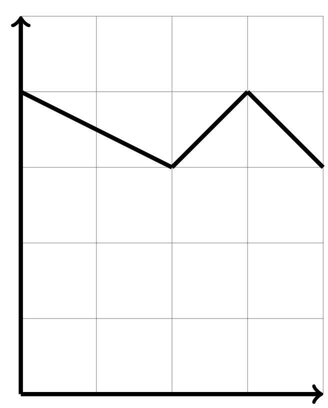
x	$(g \circ f)(x)$



Exercise 44 *Below is the portion of the graph for the function $f : \mathbb{R} \to \mathbb{R}$.*

a. *What is $f(2)$?*

b. *What is $(f \circ f \circ f)(3)$?*



Exercise 45 *You work forty hours a week at a furniture store. You receive a $220 weekly salary, plus a 3% commission on sales over $5000. Assume that you sell enough this week to get the commission. Given the functions* $f(x) = 0.03x$ *and* $g(x) = x - 5000$*, which of* $(f \circ g)(x)$ *and* $(g \circ f)(x)$ *represents your commission?*

Exercise 46 *In 2010, the Deepwater Horizon oil explosion spilled millions of gallons of oil in the Gulf of Mexico. The oil slick takes the shape of a circle. Suppose that the radius of the circle was increasing at rate .5 miles per day:* $r(t) = .5t$.

a. Let $A : \mathbb{R}_{\geq 0} \to \mathbb{R}$ be a function that describes the area of the spill in square miles t days after the spill occurred. Write a rule for $A(t)$.

b. Evaluate $A(30)$ and explain what $A(30)$ means.

Exercise 47 *According to the U.S. Energy Information Administration, a barrel of crude oil produces approximately 20 gallons of gasoline. EPA mileage estimates indicate a 2011 Ford Focus averages 28 miles per gallon of gasoline.*

a. Write an expression for $g(x)$, the number of gallons of gasoline produced by x barrels of crude oil.

b. Write an expression for $m(g)$, the number of miles on average that a 2011 Ford Focus can drive on g gallons of gasoline.

c. Write an expression for $m(g(x))$. What does $m(g(x))$ represent in terms of the context?

d. One estimate (from www.oilvoice.com) claimed that the 2010 Deepwater Horizon disaster in the Gulf of Mexico spilled 4.9 million barrels of crude oil. How many miles of Ford Focus driving would this spilled oil fuel?

8.10 Inverse Functions

Exercise 48 *$f:\mathbb{R}\to\mathbb{R}$ is given by the algebraic rule $f(x)=3x+4$. If $g:\mathbb{R}\to\mathbb{R}$ is a function with $g(7)=0$, could f and g be inverse functions? Explain.*

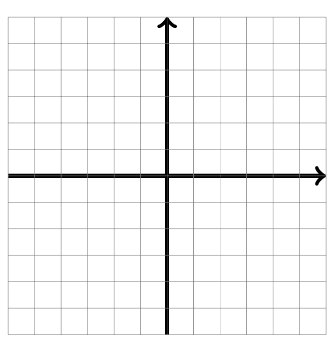
Exercise 49 *How can you tell if a function is invertible by looking at its graph? Explain your answer.*

Exercise 50 *Two functions $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ are given by algebraic rules $f(x)=\frac{3x+4}{2x-1}$ and $g(x)=\frac{x+4}{2x-3}$. Are these two functions inverses? Explain.*

Exercise 51 *Given two functions f and g does there always exists a function $f\circ g$? Explain. Give examples or counterexamples.*

Exercise 52 *$f:\mathbb{R}\to\mathbb{R}$ is given by the rule $f(x)=4x^4$.*

a. *Graph f*



b. *Is f invertible? Explain how you know.*

Exercise 53 *The following chart summarizes some values of two functions f and g.*

x	$f(x)$	$g(x)$
1	3	4
2	5	9
3	4	9
4	1	4

a. *What is the value of $f(4)$?*

b. *Evaluate $g(3)$.*

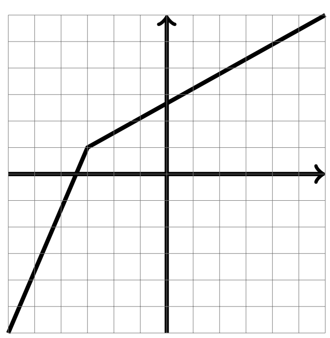
c. *Compute $(f \circ g)(1)$.*

d. *Could $g(x)$ be invertible? Explain your answer.*

e. *Is $f(x)$ invertible?*

f. *If $f(x)$ were invertible, what would the value of $f^{-1}(3)$ have to be?*

Exercise 54 *The following is a graph of a function f.*



a. *Is $f(x)$ invertible? How do you know?*

b. *What is the value of $f^{-1}(0)$?*

c. *Sketch a graph for $f^{-1}(x)$.*

8.11 Interpolating a Discrete Set of Data

Exercise 55 *You recently put* $1000 *in a bank account and earn* 5% *interest per year. Let* g *be a function such that your balance after* t *years be given by* $g(t)$.

a. Fill in the following table:

t	$g(t)$
0	
1	
2	
3	

b. Is your balance g represented by a linear function? Why or why not?

c. Let $f:[0,\infty)\to\mathbb{R}$ be a linear function given by $f(t)=1000+50t$. Fill in the following table:

t	$f(t)$
0	
1	
2	
3	

d. For some amount of time $g(t)-f(t)$ is small. For how many years would you be willing to use f to estimate g?

Exercise 56 *Rocky Mountain Power provides electricity to Salt Lake City. If you are a residential costumer of Rocky Mountain Power you pay $5.00 Basic Charge every month as well as $1.75 City Franchise Tax and a $1.28 Utah State Tax and then you are charged $0.0888540 per kwh used.*

a. Let $f(x)$ be the amount you pay using x - kwh. Notice that f is a linear function. What is the slope? What is the y-intercept?

b. If in September your electric bill is $32.39 how many kwh did you use?

c. If in September your electric bill is $32.39 what is your average cost per day?

d. If in your monthly budget you have allotted \$40.00 for the electric bill, how many kwh can you use?

e. You recently purchased a new TV you upgraded from a 32 inch TV to a 60 inch TV. Assuming average viewing, a 32 inch TV uses 60 kwh per year and 60 inch TV uses 165 kwh per year. How much would you expect your monthly bill to go up?

f. When you enroll in Blue Sky, Rocky Mountain Power purchases certified renewable energy certificates from regional renewable energy facilities on your behalf. This guarantees that electricity from wind facilities is delivered to the regional power system. Electricity from renewable energy facilities replaces and reduces the need for electricity generated from non-renewable sources like fossil fuels, creating measurable environmental benefits. If you buy one block of renewable energy it costs \$1.95. Would buying a block of renewable energy effect the slope of the cost function or the y-intercept? (write m or b)

Exercise 57 *If a computer program has a loop in it, the length of time it takes the computer to run the program varies linearly with the number of times it must go through the loop. Suppose a computer takes 8 seconds to run a given program when it goes through the loop 100 times, and 62 seconds when it loops 1000 times.*

a. Write the particular equation expressing seconds in terms of loops.

b. Predict the length of time needed to loop 30 times; 10,000 times.

c. Suppose the computer takes 23 seconds to run the program. How many times does it go through the loop?

d. How long does it take the computer to run the rest of the program, excluding the loop? What part of the mathematical model tells you this?

e. How long does it take the computer to go through the loop once? What part of the mathematical model tells you this?

f. Plot the graph of this function.

8.12 Slope

Exercise 58 *Below is a table for a function $f : \mathbb{R} \to \mathbb{R}$. Could f be a linear function? Why or why not?*

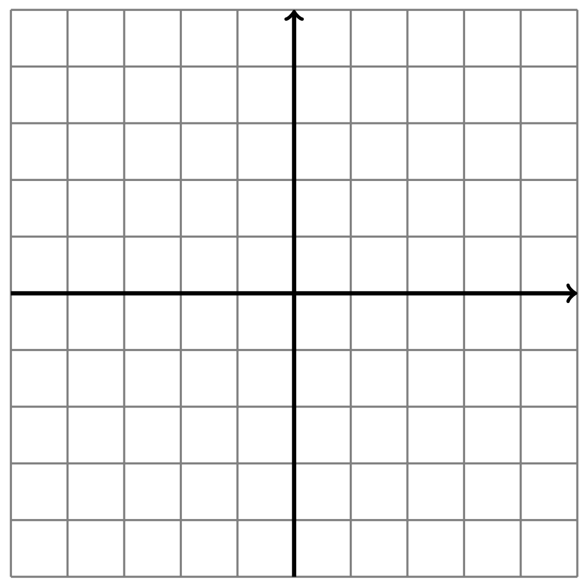
x	$f(x)$
3	14
4	17
5	21

Exercise 59 *Find the slope between the following pairs of points:*

a. $(-1,3)$ and $(2,7)$

b. $(2,6)$ and $(5,10)$

c. $(2,-4)$ and $(5,0)$

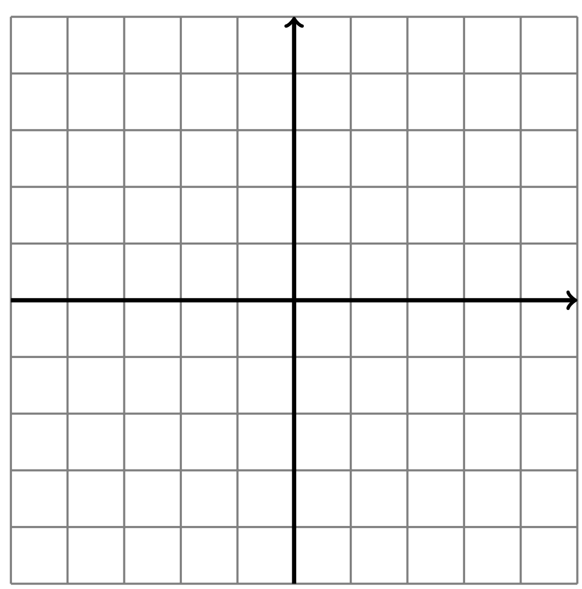


Once finished, graph all the pairs and the line segments between them. What do you notice?

Exercise 60 *Draw the line segments between the following pairs of points:*

a. $(0,0)$ and $(2,7)$

b. $(0,0)$ and $(-7,2)$

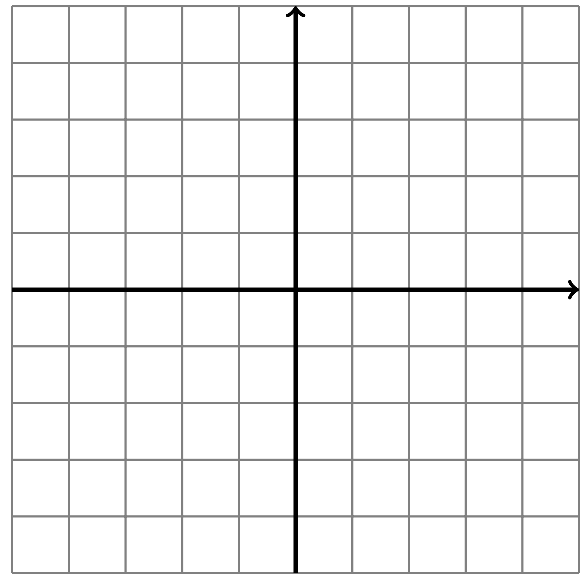


What is the slope of each of the line segments?

Exercise 61 *Draw the line segments between the following pairs of points:*

a. $(-1,3)$ and $(2,7)$

b. $(1,-3)$ and $(-3,0)$



What is the slope of each of the line segments?

Exercise 62 *Below is a table for an arithmetic sequence:*

x	$f(x)$
1	-5
2	-2
3	1

Can this table represent a linear function as well?

What is the explicit formula for the n^{th} term of this sequence?

8.13 Lines

Exercise 63 *Let $f : \mathbb{R} \to \mathbb{R}$ be a function given by the algebraic rule $f(x) = 4x + 5$.*

a. What is the $x-$intercept?

b. What is the $y-$intercept?

c. What is the slope?

d. For what value of x does $f(x) = 17$?

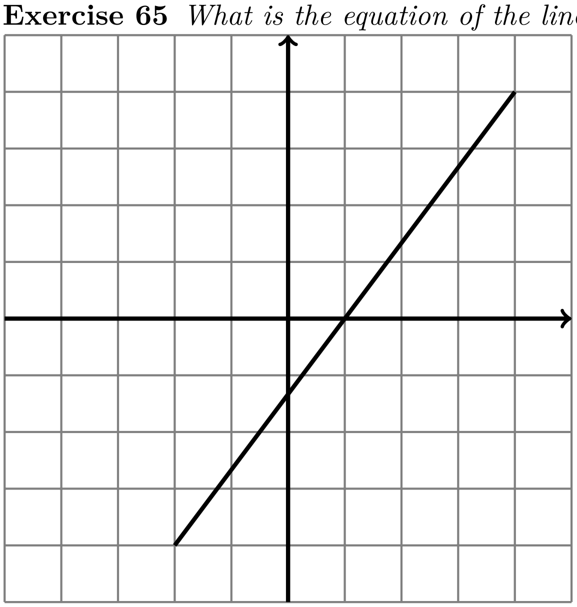
Exercise 64 *Let $g : \mathbb{R} \to \mathbb{R}$ be a function given by the algebraic rule $g(x) = -2x + 4$.*

a. What is the x-intercept?

b. What is the y-intercept?

c. For what value of x does $g(x) = 12$?

Exercise 65 *What is the equation of the line graphed below:*



Exercise 66 *What is the equation for the line that goes through the points $(3,4)$ and $(2,5)$? Report the slope and the $y-$intercept.*

Exercise 67 *Is the point* $(2,1)$ *on the line given by* $3y+2x=7$*? How do you know?*

Exercise 68 *Let* $f:\mathbb{R}\to\mathbb{R}$ *be a function given by* $f(x)=4x+3$*.*

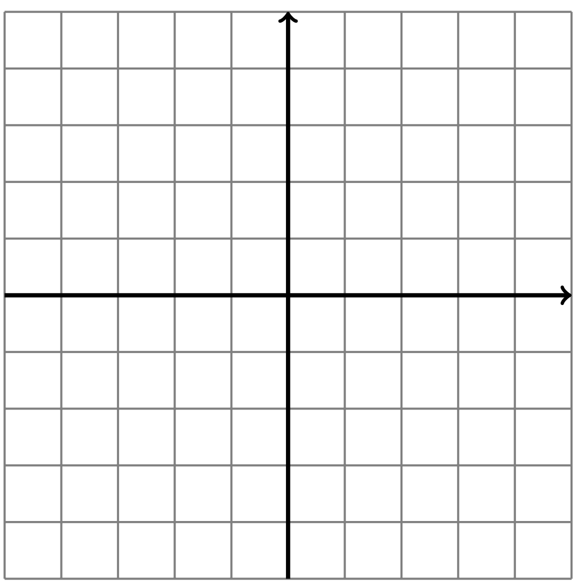
a. Is the graph of f is a line? How do you know?

b. What is the x-intercept?

c. What is the y-intercept?

d. What is the slope?

e. Sketch the graph for f. Which quadrant does the graph not pass through?



Exercise 69 *Which of the following functions from* $\mathbb{R}$ *to* $\mathbb{R}$ *are linear?*

$f(x)=x\sqrt{2}-\frac{1}{2}$ Yes No

$2x+\frac{y}{4}=2$ Yes No

$y=\frac{2}{x}+2$ Yes No

$y-x+1=0$ Yes No

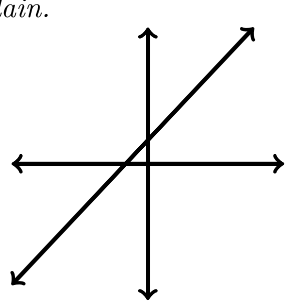
Exercise 70 *Does the point* $(8,1)$ *lie on the line* $y = -\frac{1}{2}x + 3$*? Show how you arrived at the answer.*

Exercise 71 *Find a pair* (x,y) *that satisfies the equation* $y = -\frac{1}{2}x + 3$.

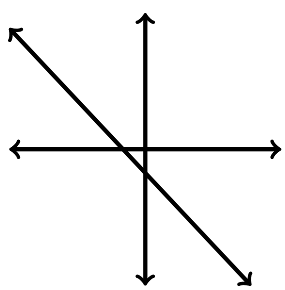
Exercise 72 *Does the pair you found in the previous question lie on the line* $y = -\frac{1}{2}x + 3$*? Explain.*

Exercise 73 *Which of the following graphs could possibly be a graph of* $y = -\pi x + \sqrt{73}$*? Explain.*

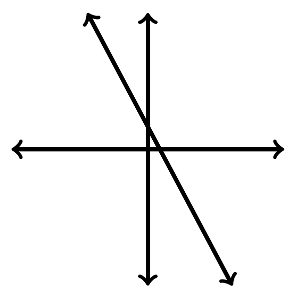
A



B

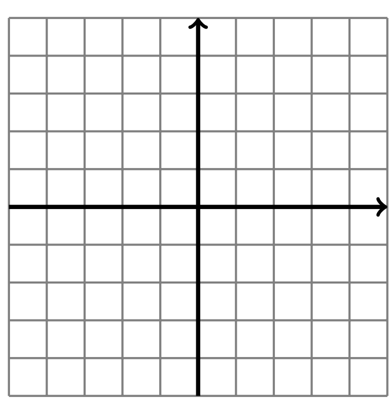


C



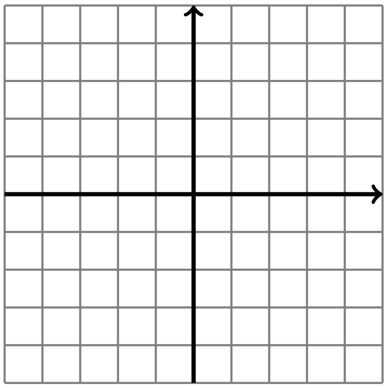
Exercise 74 *Given the equation* $y = 3x - 7$*:*

a. Evaluate y if $x = -1, x = 2$, and $x = 5$.



b. Show by graphing that the points lie in a straight line.

Exercise 75 *Plot the graph of $y+3=-2(x+1)$. Then transform the equation to slope-intercept form. Transform the equation to $Ax + By = C$ form, where A, B, and C are integers.*



Exercise 76 *Find the particular equation of the line described.*

a. Contains $(2,-7)$ and $(5,3)$.

b. Contains $(-4,1)$ and is parallel to the graph of $2x-9y=47$.

c. Contains $(3,-8)$ and is perpendicular to the graph of $y=0.2x+11$

d. Has x-intercept of 5 and y-intercept of -6.

e. Is vertical, and contains $(-13,8)$.

f. Is horizontal, and contains $(22,\pi)$.

g. Has the x-axis as its graph.

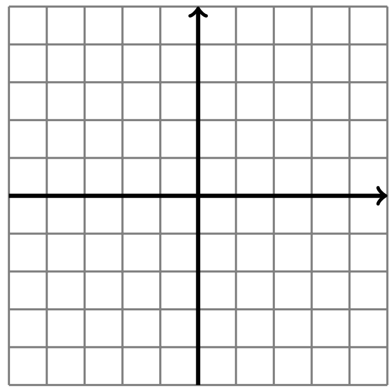
h. Has a slope that is infinitely large, and contains $(5,7)$.

Exercise 77 *Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be functions given by the following algebraic rule $f(x)=4x+5$ and $g(x)=-2x+4$.*

a. Is there a value of x such that $f(x)=g(x)$?

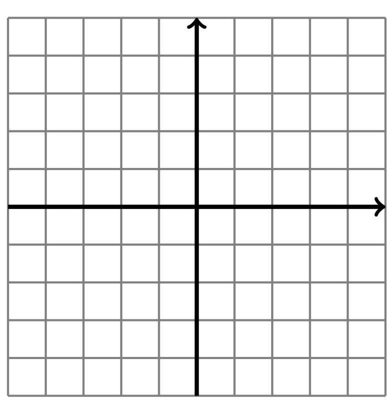
b. How many values of x does $f(x)=g(x)$?

c. What does this value of x tell us about the graphs of f and g?



Exercise 78 *Solve the following two equations at the same time, that is find a pair of numbers: x and y which make both equations true*

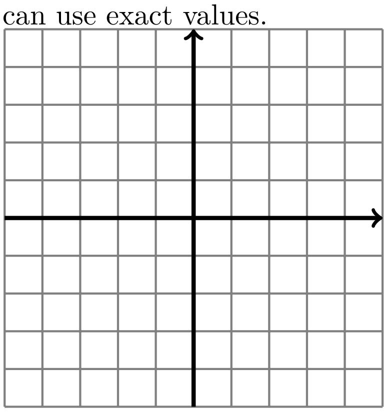
$$\begin{aligned} y &= 3x+5 \\ 2x-y &= 4. \end{aligned}$$



Exercise 79 *The following table records the prices for Horizon Organic Fat-Free Milk at Target (1110 S 300 W).*

Gallons	Price
.25	\$2.49
.5	\$3.99
1	\$6.99

For the purposes of this problem, you may round the prices to the nearest tenth, or you can use exact values.



a. Does (Gallons,Price) lay on a line? How do you know?

b. Write the equation of the line that passes through at least two of the points given.

c. Use the equation you developed to determine the price of 10 gallons of milk.

d. If you paid \$14.50 for milk, according to your linear function how many gallons of milk did you buy?

8.14 Graphs of Quadratic Functions

Exercise 80 *If $y = ax^2 + bx + c$, where a, b, c are constants and x, y are variables answer the following questions.*

a. What does the graph look like?

b. When is the parabola opening up?

c. Opening down?

d. When does the graph have a maximum value?

e. Minimum value?

f. Will it always have x-intercepts?

g. Will it always have a y-intercept?

Exercise 81 *Graph the following functions:*

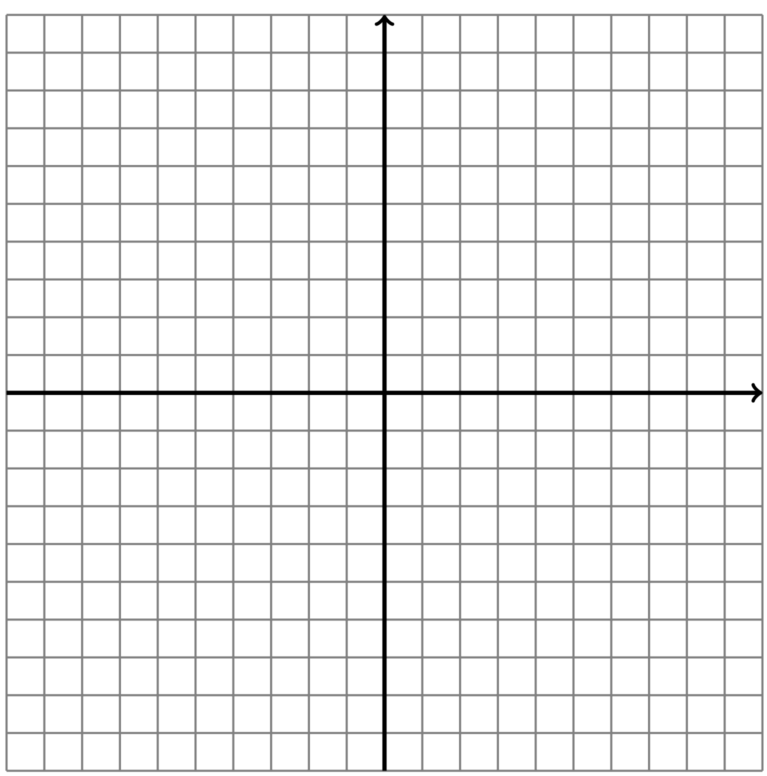
a. $f(x) = (3x - 1)(x + \sqrt{12})$

b. $g(x) = (x - 3)(x + 3) - 2$

c. $h(x) = x^2 - 2x$

d. $m(y) = y^2 - 16$

e. $p(c) = c^2 - 12 - c$



Exercise 82 *Each table represents either a linear, quadratic or "neither" function. Identify whether each table is linear, quadratic, or neither, and then* **write a sentence explaining how you know**.

1. Table A

x	$f(x)$
-1	1.5
0	2
1	3
2	5

Table A is (circle one):
Linear · **Quadratic** · **Neither**

because:

2. Table B

x	$f(x)$
-1	4
0	3
1	4
2	7

Table B is (circle one):
Linear · **Quadratic** · **Neither**

because:

3. Table C

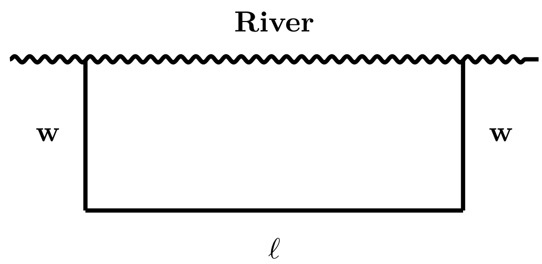
x	$f(x)$
-1	3
0	1
1	-1
2	-3

Table C is (circle one):
Linear · **Quadratic** · **Neither**

because:

Exercise 83 *Find the x-intercepts, and the y-intercept of the graph whose equation is $y = x(x-7)$.*

Exercise 84 *What if we have 50 feet of fencing but we can build Ellie's pen next to a river so that we only need to enclose 3 sides as in the picture below.*



a. Make a table for values of ℓ and w.

b. Graph w as a function of ℓ.

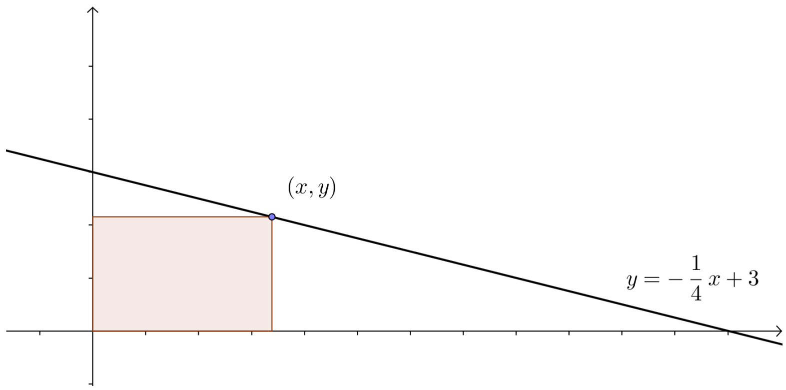
c. Write the width w as function of the length ℓ.

d. Make a table of ℓ and Area.

e. Write area as a function of ℓ.

f. What choice of ℓ maximizes the area?

Exercise 85 *A point from the first quadrant is chosen on the line $y = -\frac{1}{4}x + 3$. Lines are drawn from this point parallel to the axes so that a rectangle is formed under the given line (as in the figure).*



a. Among all such rectangles, what are the dimensions of the one with the maximum area?

b. What is the maximum area?

8.15 The Zero Product Property

Exercise 86 *What does it mean to "solve" an equation?*

Exercise 87 *How do you draw a picture to show the factorization of a quadratic expression? What do the factors tell you geometrically? Where is the expression in standard form in your picture?*

Exercise 88 *Factor the following quadratic expressions (given to you in standard form; your job is to rewrite the expressions in factored form) by drawing a picture:*

a. x^2+8x+7

b. $x^2+8x+15$

c. $x^2+8x+16$

d. x^2+4x+3

e. $2x^2+4x+2$

Exercise 89 *In this question you will think about the differences and similarities between expression, equation, and function.*

a. Factor $x^2 + 12x + 36$.

b. Solve $x^2 + 12x + 36 = 0$.

c. Graph the function $f(x) = x^2 + 12x + 36$. What are the x-intercepts? What are the y-intercepts? What is the axis of symmetry? Does f have a minimum or a maximum? What is the vertex?

d. Factor $x^2 + 8x + 15$.

e. Solve $x^2 + 8x + 15 = 0$ for x.

f. Graph $f(x) = x^2 + 8x + 15$. What are the x-intercepts? What are the y-intercepts? What is the axis of symmetry? Does f have a minimum or a maximum? What is the vertex?

Exercise 90 *If $a \cdot b = 0$, what can you say about a or b? This question will assess your ability to use the zero product property.*

a. Solve the equation $(3x+3)(4x+5) = 0$ for x. How did you accomplish this task?

b. Could you solve $x+4$?

c. Could you solve $(x+4)(x-4)(x+1) = 0$?

d. Multiply: $(x+4)(x-4)(x+1)$. What is the degree of the product?

e. If you graph $f(x) = (x+4)(x-4)(x+1)$, what would the x-intercepts be? What would the y-intercepts be?

Exercise 91 *Write a quadratic equation, in the form $ax^2 + bx + c = 0$, whose roots are 2 and 5.*

Exercise 92 *Solve the following equations:*

1. $(3x-1)(x+\sqrt{12})=0$

2. $(x-3)(x+3)=2$

3. $x^2-2x=0$

4. $y^2-16=0$

5. $c^2-12=c$

6. $\frac{x}{9}=\frac{4}{x}$

8.16 Completing the Square

Exercise 93 *Solve the following quadratic equations:*

a. $x^2 - x - 6 = 0$

b. $x^2 - 7x + 12 = 0$

c. $x^2 + 16x - 17 = 0$

d. $x^2 + 29x + 100 = 0$

e. $x^2 + 8x = -7$

f. $x^2 + 8x + 15 = 0$

g. $x^2 = -8x - 16$

h. $x^2 + 3x + 1 = 0$

i. $2x^2 + 3x + 1 = 0$

j. $x^2 + 4x + 3 = 0$

k. $2x^2 + 4x = -2$

l. $(2x + 3)(x - 1) = 0$

m. $(x+3)^2 = 2$

n. $(x+3)^2 = 2x-7$

o. $-9(x-5)(x+2) = 0$

p. $x^2+3x+2 = 0$

q. $x^2-3x+2 = 0$

Exercise 94 *Solve the following quadratic equation using the completed square form:*
$9x^2+4x+2=0$

Exercise 95 *The square of a number exceeds 5 times the number by 24. Find the number(s).*

Exercise 96 *Looking back at the entire chapter on quadratic functions, answer the following essential questions:*

a. What are the x-intercepts?

b. For a general quadratic function $g : \mathbb{R} \to \mathbb{R}$ how do you find the x-intercepts?

c. What are the y-intercepts?

d. For a general quadratic function $g : \mathbb{R} \to \mathbb{R}$ how do you find the y-intercepts?

e. Let g be a general quadratic function. How many y-intercepts can g have? How many x-intercepts can g have?

f. What is the axis of symmetry? How did you know this?

g. In general what is symmetry?

h. What does the sign of the x^2 term in $g(x)$ tell you about its graph?

i. What is the maximum or minimum value?

j. How can we find the zeros of a quadratic function?

k. How do we calculate the max or min of a quadratic function?

Notes:

8.17 Rules for Exponents

Exercise 97 *Rewrite the following using one exponent. Justify your answers.*

1. a^5a^6
2. $(a^4)^5$
3. $(5^5)^2 5^3$
4. $-5y^2y^3$
5. $3z \cdot z^3$
6. $3t^2 \cdot (-2t^6)$
7. $(vw^3)^5 \cdot v^2$
8. $3y^3 \cdot 2y^2$
9. $(-3t^4)^3$
10. $q^3p^2 \cdot (p^2q^3)^4$
11. $(-5pr)(r^2p^3)^3$
12. $a \cdot a^2 \cdot (-a)^4$
13. $(t^3r^2)^3$

Exercise 98 *Rewrite the following expressions using one exponent:*

1. $(a^5a^{-3})^3$
2. $(a^{-2}a^3)^2$
3. $(\frac{x}{x^2})^3$

Exercise 99 *Explain how the exponent* $^{-1}$ *means different things when we think about* a^{-1} *and* f^{-1}*, where* a *is a real number, and* f *is a function.*

Exercise 100 *Rewrite the following using positive exponents. Justify your answers.*

1. 5^{-1}
2. 7^{-3}
3. $\frac{1}{4^{-2}}$
4. $\frac{3^4}{3^5}$
5. 8^{-2}
6. $\frac{27m^5n^6}{9mn^3}$
7. $\frac{28x^2y^3}{2xy^2}$
8. $(\frac{3x}{4y})^2$
9. $\frac{5x^2y}{10x^4y^2}$
10. $\frac{7t^2}{tr^3} \cdot \frac{2tr^5}{3r}$
11. $\frac{(-4xy)^3}{8xy^2}$
12. $(\frac{7}{8})^0$
13. $\frac{5}{xy^{-1}}$
14. $\frac{5}{5^{-2}}$

Exercise 101 *Give an example to show how the Quotient of Powers rule can explain why* $b^0 = 1$.

Exercise 102 *Give an example to show how the Quotient of Powers rule can explain why* $b^{-2} = \frac{1}{b^2}$.

Exercise 103 *The Earth is about* $93 \cdot 10^6$ *miles from the sun. Light travels at about* $1.86 \cdot 10^5$ *miles/second. About how long does it take light from the sun to reach the Earth? Show how one or more of the laws of exponents is useful in solving this problem.*

Exercise 104 *In 1980, the population of the U.S. was about* $227 \cdot 10^6$. *If the land of the U.S. was about* $35 \cdot 10^6$ *square miles, what was the average number of people per square mile of land? Show how one or more of the laws of exponents is useful in solving this problem.*

Exercise 105 *For which figure is the ratio of the volume to surface area greater: a sphere or a cube? (Volume of a sphere:* $\frac{4}{3}\pi r^3$, *where* r = *radius; Surface area of a sphere:* $4\pi r^2$*)*

Notes:

8.18 Graphs of Exponential Functions

Exercise 106 *What can you say about the shape of the graph of $f(x) = ab^x$ if $a > 0$ and $0 < b < 1$? As b increases in value from 0 toward 1 how does the graph change?*

Exercise 107 *What can you say about the shape of the graph of $f(x) = ab^x$ if $a > 0$ and $b > 1$? As b increases in value how does the graph change?*

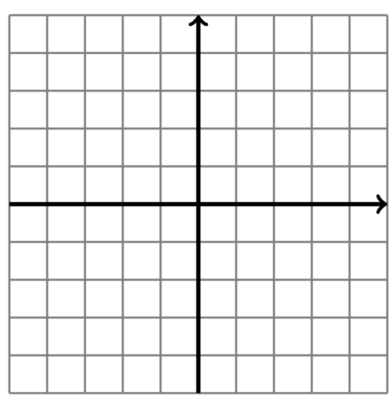
Exercise 108 *Suppose $b = 1$ in the equation $f(x) = ab^x$, where $a > 0$. What does the graph of this function look like? In what way(s) is this graph fundamentally different from the graphs you sketched in the previous two problems? Answer the same question for when $b = 0$.*

Exercise 109 *Now suppose $b = -2$ and $a = 1$, so the equation is $f(x) = (-2)^x$.*

a. Without using the graphing or table features of your calculator complete the following table. [1]

x	-2	-1	0	1	2	3	4
$f(x)$							

b. Plot the points you found in Part a and connect them with a smooth curve so as to indicate what the graph of $f(x) = (-2)^x$ might look like.



c. Describe how the graph in Part b differs from all the other graphs you've sketched.

d. Explain why we do not use negative numbers for b when we speak of exponential functions $f(x) = ab^x$.

[1] Make sure to enclose -2 in parentheses when doing your calculations.

Exercise 110 *Determine the value of a and b in $f(x) = ab^x$ if the following facts are known about f:*

a. $(0,3)$ *and* $(2,5)$ *are on the graph of the function.*

b. $(1,2)$ *and* $(3,10)$ *are on the graph of the function.*

c. $(2,3)$ *and* $(8,1)$ *are on the graph of the function.*

d. $(-3,8)$ *and* $(1,1)$ *are on the graph of the function.*

e. $f(0) = 5$ *and* $f(3) = 20$.

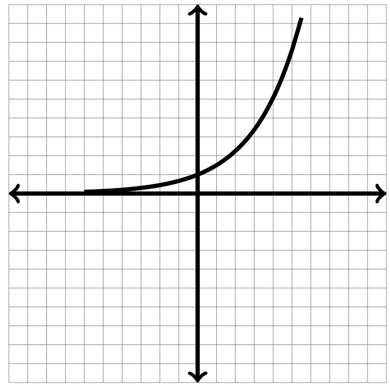
Exercise 111 *The graph of $y = f(x)$ is shown below. Use this graph to quickly sketch a reasonable graph for each of the following functions:*

a. $y = f(x) + 2$

b. $y = f(-x)$

c. $y = -f(x)$

d. $y = -f(x) + 2$



Notes:

8.19 Inverse Functions

Exercise 112 *Evaluate each of the following logarithms:*

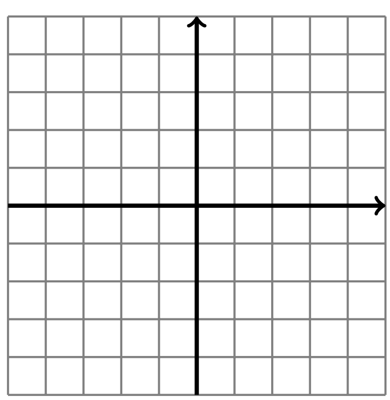
a. $\log_{10} 1000$

b. $\log_{10} \frac{1}{100}$

c. $\log_3 \frac{1}{3}$

d. $\log_3 \sqrt{3}$

Exercise 113 *Graph the inverse function of $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 5^x$. Make sure that you clearly state the domain and the target of f^{-1}.*



Exercise 114 *Explain why there is no function given by the rule $f(x) = \log_{-3}(x)$.*

Exercise 115 *A function $g : \mathbb{R} \to \mathbb{R}$ is given by the rule $g(x) = 15 \cdot \left(\frac{1}{2}\right)^x - 3$. Decide whether g has an inverse function. If it does, what is the algebraic expression for the inverse?*

Exercise 116 *Find the expressions for the inverse functions of the following functions:*

a. $a(x) = 3\log_{10} x - 1$

b. $b(x) = \log_{10} x^2 - 2$

c. $c(x) = \frac{1}{4} \cdot 3^{2x-1}$

d. $d(x) = \frac{1500}{2+3^{x-1}}$

8.20 Solving Exponential and Logarithmic Equations

Exercise 117 *Solve the following exponential equations:*

a. $4^{x-1} = 16$

b. $7^{2x+1} = 7^{3x-1}$

c. $3^{2x-1} = 27^{x}$

d. $5^{3x-8} = 25^{2x}$

Exercise 118 *Solve the following exponential equations:*

a. $5^{x} = 9$

b. $3^{2x+1} = 15$

c. $\left(\frac{1}{2}\right)^{x} = 3$

d. $e^{x} = 30$

e. $12 \cdot 5^{0.1x} = 30$

f. $3^{x-1} - 4 = 7$

Exercise 119 *Solve the following logarithmic equations:*

a. $\log_4 x = 2$

b. $\log_4(x+2)x = 2$

c. $3\log_7(x-1) = 6$

d. $\frac{2}{3}\log_4(x-1)+3 = 6$

Exercise 120 *Looking back at the entire chapter on exponential functions, answer the following essential questions:*

a. Why are exponents useful? How are they used in real world applications?

b. What does a negative exponent mean? What does a fractional exponent mean?

c. Where do rules for exponents come from?

8.21 Applications

Exercise 121 *Disease can spread quickly. Suppose the spread of a direct contact disease in a small town is modeled by the function:*

$$P(t) = \frac{10000}{1 + 2^{3-t}}$$

where $P(t)$ is the total number of people infected after t days.

a. Estimate the initial number of people infected with the disease. Explain how you found your answer.

b. How many people will be infected after 1,2,3,4, and 5 days? (Fill out a table)

c. What is the maximum number of people who can become infected? How do you know?

d. The town officials must inform the citizens when 5000 people become infected. Which day will the town officials make an announcement?

Exercise 122 *As soon as you drive a new car off the dealer's lot, the car is worth less than what you paid for it. This is called depreciation. Chances are that you will sell it for less than the price that you paid for it. Some cars depreciate more than others, but most cars do depreciate. On the other hand, some older cars actually increase in value. This is called appreciation. Suppose you have a choice between buying a 1999 Mazda Miata for $19,800 which depreciates at 22% a year, or a 1996 Honda Civic EX for $16,500 which only depreciates at 18% a year. In how many years will their values be the same? Should you instead buy a 1967 Ford Mustang for $4,000 that is appreciating at 10% per year? Which car will have the greatest value in 4 years? In 5 years?*

Exercise 123 *Suppose that the number of bacteria per square millimeter in a culture in your biology lab is increasing exponentially with time. On Tuesday there are 2000 bacteria per square millimeter. On Thursday, the number has increased to 4500 per square millimeter.*

a. Derive the particular equation.

b. Predict the number of bacteria per square millimeter that will be in the culture on Tuesday next week.

c. Predict the time when the number of bacteria per square millimeter reaches 10,000.

d. Draw the graph of the function.

Exercise 124 *If you invest* \$1000 *at* 4% *interest and it is compounded continuously, your balance is given by the function* $b(t) = 1000(2.718)^{.04t}$ *(where* t *is the number of years you have invested your money).*

a. How much money do you have after 1 year?

b. How much money do you have after 4.5 years? What does the fractional exponent mean?

c. How long would you have to wait to have \$10,000?