Warming up for the semester - Section 1 Introduction.

Instructor: Brendan



- ▶ Turn to your neighbor whom you do not already know
 - ▶ Tell them your name.
 - ▶ Tell them what you do when you are free to do what you want.
 - ▶ Tell them why you are here.

Warming up for the semester - Section 1

▶ The person you just met will be your partner through the day.

▶ Work on the first 4 problems from the handout for the next 10 minutes with your partner.

▶ Be ready to share whatever progress you made as a team.

(Brazil) Two mothers and two daughters sleep in the same room. There are only three beds and exactly one person sleeps on each of them, yet all people are accounted for. How is this possible?

(Ireland) One day three brothers were going past a graveyard. One of them said, "I shall go in so that I may say a prayer for the soul of my brother's son." The second man said the same thing. The third brother said, "I shall not go in. My brother's son is not there." Who is buried in the graveyard?

(Puerto Rico) Who is the sister of my aunt, who is not my aunt, but is the daughter of my grandparents?

(Russia) An old man was walking with a boy. The boy was asked, "How is the old man related to you?" The boy replied, "His mother is my mother's mother-in-law. What relation is that?

Warming up for the semester - Section 1.3 Clicker Question

Was that mathematics?

- 1. Yes.
- 2. No.

Warming up for the semester - Section 1.3 Clicker Question

Is this more like mathematics?

- 1. Yes.
- 2. No.
- Solve for x: x + 8 = 9.

• Solve for y: y + 10 = 11.

• Solve for z: z + 12 = 12.

Warming up for the semester - Section 1



Warming up for the semester - Section 1 Our goals for this class

Overarching Goals:

- Problem solving
- Reasoning

Communicating

Content Goals:

- Find and describe patterns and relationships
- Analyze and use these to make predictions
- Speak in general rather than in particular
- Learn how to use variability productively

Warming up for the semester - Section 1.3 To achieve this we need a different kind of class

Active engagement in class

- ▶ Working in class
- Collaboration
- Clickers
- ▶ Free of distractions

Active engagement outside of class

- Weekly homework
- Projects
- Independent studying
- Discussion board

Warming up for the semester - Section 1.3 What will the class look like?

Class

- Partner work/problem solving
- Discussion sessions
- Mini lectures
- ▶ Independent work/clicker questions
- Quizzes/writing prompts

Post-class

- ▶ Look through the work done to locate points of confusion
- One or two questions to solidify the work possible
- ▶ Procedural, weekly homework ongoing
- ▶ Regular, biweekly homework ongoing

Warming up for the semester - Section 1.3 Available Support

- ► Office hours:
 - ▶ JWB 115
 - ▶ MW 10:45 11:45 and by appointment.
- ▶ T. Benny Rushing Mathematics Student Center



Warming up for the semester - Section 1.3 Contract.

▶ Take 5 minutes to quietly read and fill out the contract.

▶ Exchange with your partner. Have them read it and sign it as well.

▶ I will look them over and sign them.

Warming up for the semester - Section 1.3 Things you will need.

- Clicker
- Course packet
- Extra paper
- Writing utensil
- Scientific calculator

Warming up for the semester - Section 1.3 Grades.

Assessment Tool	Value	
Attendance and Participation	10%	Clickers
Quizzes	15%	Weekly
Written Homework	15%	Frequent
Midterms	30%	Three total
Final Exam	30%	You have to take the
		final to pass the course!

Warming up for the semester - Section 1.3 Comments. Questions. Complaints.

Warming up for the semester - Section 1.3 Back to work.

Work on Questions 1.5 - 1.7. You may want to use a calculator.

- ▶ Put in first 3 digits of your phone number
- ▶ Multiply by 80
- ▶ Add 1
- ▶ Multiply by 250
- ▶ Add the last four digits of your phone number
- ▶ Add the last four digits of your phone number
- ▶ Subtract 250
- ▶ Divide by 2
- ▶ What did you get?

Was that surprising? Try to explain why that happened.

Warming up for the semester - Section 1.3 Question 1.5 - Click in

What did you get for your phone number question? 1. 777

2. Magical things happened!

3. Don't be silly. There is no magic. It is all perfectly sensible.

4. I do not get it, what is the big deal?

What happened?

A pot and a lid cost \$11 (this was once upon a time). The pot costs \$10 more than the lid. How much does each item cost individually?

What do these questions have to do with mathematics? What do they have to do with algebra? Describe the process you used to solve these questions.

Sequences - Section 2 Mathematics investigates patterns.

Essential Questions.

▶ How do we describe a pattern?

▶ How can patterns be used to make predictions?

▶ What are some ways to represent, describe, and analyze patterns?



- 1. Describe the pattern that you see in the sequence of figures above.
- 2. Assuming the sequence continues in the same way, how many dots are there on the fourth day? On the fifth day? On the tenth day?
- 3. How many dots are there on the 100^{th} day?



1. Describe the pattern that you see in the sequence of figures. Visual Patterns - Section 2.2 Question 2.1 - Click in

How many dots on the tenth day?

1. 40

2. 37

3. 36

4. 41



2. Assuming the sequence continues in the same way, how many dots are there on the fourth day? On the fifth day? On the tenth day?



3. How many dots are there on the 100^{th} day?

Look at the pattern below and answer the questions:



- 1. Describe the pattern that you see in the sequence of figures above.
- 2. Assuming the sequence continues in the same way, how many dots are there on the fourth day? On the fifth day? On the tenth day?
- 3. How many dots are there on the 100^{th} day?



1. Describe the pattern that you see in the sequence of figures.



2. Assuming the sequence continues in the same way, how many dots are there on the fourth day? On the fifth day? On the tenth day?

How many dots on the 100^{th} day?

1. 402

2. 602

3. 406

4. 604



3. How many dots are there on the 100^{th} day?

Tiling a Pool - Section 2.3 Question 2.3

The summer season is nearly over and the owner of the local pool club is thinking of what all needs to be done once the pool closes. One of the common things in need of repair are the tiles around the perimeter of the pool. In the picture below a 5 foot square pool has been tiled with 24 square tiles (1 foot by 1 foot).



Tiling a Pool - Section 2.3 Question 2.3

- 1. Make sketches to help you figure out how many tiles are needed for the borders of square pools with sides of length 1, 2, 3, 4, 6, 10 feet without counting. Record your results in a table.
- 2. Write an equation for the number of tiles N needed to form a border for a square pool with sides of length s feet. How do you see this equation in the table? How do you see the equation in your pictures?
- 3. Try to write at least one more equation for N. How would you convince someone that your expressions for the number of tiles are equivalent?
- 4. Use your work to decide how many tiles you would need for a square pool whose sides are 127 feet long. What about a square pool whose sides are 128 feet long?
- 5. Graph the relationship you observed between s (the side length) and N (the number of tiles needed).
> Make sketches to help you figure out how many tiles are needed for the borders of square pools with sides of length 1, 2, 3, 4, 6, 10 feet without counting. Record your results in a table.

1. Record your results in a **table**.

s	N
side length	number of tiles
1	
2	
3	
4	
5	24
6	
10	

2. Write an equation for the number of tiles N needed to form a border for a square pool with sides of length s feet. How do you see this equation in the table? How do you see the equation in your pictures?

> 3. Try to write at least one more equation for N. How would you convince someone that your expressions for the number of tiles are equivalent?

Tiling a Pool - Section 2.3 Question 2.3 - Click in.

How many tiles you would need for a square pool whose sides are 127 feet long?

- 1. 508
- **2**. 524
- **3**. 512
- 4. 135

> 4. Use your work to decide how many tiles you would need for a square pool whose sides are 127 feet long. What about a square pool whose sides are 128 feet long?

5. Graph the relationship you observed between s (the side length) and N (the number of tiles needed).

s	N
1	8
2	12
3	16
4	20
5	24
6	28
10	44



6. Relate the growth pattern in each of the representations of the pattern (table, equation, graph).

People describe the number of tiles differently. Show their solution in the diagram and explain their thinking.





4(s+1)



4(s+1)



4s + 4



2s + 2(s+2)



4(s+2) - 4



 $4(s+2\cdot\frac{1}{2})$



$$(s+2)^2 - s^2$$

How can you convince someone that all of the expressions are equivalent? Use both properties of operations on whole numbers as well as the diagrams.

Tiling a Pool - Section 2.3 Sequence

What does the word "sequence" mean?

- 1. Take 1 minute to write down your response.
- 2. Exchange with your neighbor.
- 3. How are your definitions the same? How are they different?



Se · quence

noun

- a particular order in which related events, movements, or things follow each other. "the content of the program should follow a logical sequence" synonyms: succession, order, course, series, chain, train, string, progression, chronology, timeline; More
- a set of related events, movements, or things that follow each other in a particular order.

"a grueling sequence of exercises"

verb

- arrange in a particular order. "trainee librarians decide how a set of misfiled cards could be sequenced"
- 2. play or record (music) with a sequencer.

Tiling a Pool - Section 2.3 Sequence

Definition

An infinite list of numbers is called a **sequence**. Sequences are written in the form

 a_1, a_2, a_3, \ldots

 a_n is called the **nth** term of the sequence.

Tiling a Pool - Section 2.3 Sequence

Did we have a sequence in this problem?

If p_1, p_2, p_3, \ldots is a sequence such that

 $p_n = \#$ tiles around a square pool of side length n,

- 1. What is the value of p_5 ?
- 2. What is the value of p_{15} ?
- 3. What is the value of p_n ?
- 4. What is the relationship between p_n and p_{n+1} .

1. What is the value of p_5 ?

2. What is the value of p_{15} ?

3. What is the value of p_n ?

4. What is the relationship between p_n and p_{n+1} .

Tiling a Pool - Section 2.3 Arithmetic Sequence

Definition

A sequence a_1, a_2, a_3, \ldots is an **arithmetic sequence** if there is a number d such that you obtain any member of the sequence by adding d to the member that came before it. Symbolically, we'd write that:

$$a_n = a_{n-1} + d.$$

Is the sequence p_1, p_2, p_3, \dots (from Question 2.5) an arithmetic sequence?

Tiling a Pool - Section 2.3 Arithmetic Sequences

Which of the following are arithmetic sequences?

1.
$$2, 7, 12, 17, 22, \dots$$

2. $3, 9, 27, 81, \dots$
3. $5, 8, 11, 14, 17, 20, 23, 26, 29, 32, \dots$
4. $1, -1, 1, -1, 1, -1, \dots$
5. $1, 3, 5, 7, 9, 11, 13, 15, \dots$

6. $4, 9, 16, 25, 49, \ldots$

Tiling a Pool - Section 2.3 Arithmetic Sequences - Click In!

Is the following an arithmetic sequence:

 $19, 13, 7, 1, -5, \ldots$

1. Yes

2. No

Tiling a Pool - Section 2.3 Arithmetic Sequences - Click In!

The following is an arithmetic sequence:

$$7, 2, -3, \ldots$$

What is the next term of the sequence?

- 1. -8
- 2. 8
- 3. -5
- 4. Other

Geometric Sequences - Section 2.4 Question 2.7

Work on Question 2.7 with your neighbor.



Geometric Sequences - Section 2.4 Question 2.7

Social media has created a way to quickly share information (articles, videos, jokes, ...). Gangnam Style is a YouTube video that became popular in July 2012. On September 6^{th} , the video had 100,000,000 views. On December 21^{st} the video was the first video in history to have over 1,000,000,000 views. If Gangnam style was released on July 15, how many days did it take to for the video to hit 100,000,000 views? How many days did it take for the video to breach 1,000,000,000 views?

Geometric Sequences - Section 2.4 Question 2.7 - Click In!

How many days did it take to reach 100,000,000 views?

- 1. 52
- 2. 53
- **3**. 51
- **4**. 54

Geometric Sequences - Section 2.4 Question 2.7 - Click In!

How many days did it take to reach a billion views?

- 1. 155
- 2. 157
- **3**. 169
- 4. 161

Geometric Sequences - Section 2.4 Question 2.8

To model the sensation of "viral videos", assume that on day one there was one view, that every new view corresponds to a new person seeing the video and on average a new viewer shows the video to 2 new people.

- 1. How many times was the video viewed on day 2?
- 2. How many times was the video viewed on day 3?
- 3. How many times was the video viewed on day 5?
To model the sensation of "viral videos", assume that on day one there was one view, that every new view corresponds to a new person seeing the video and on average a new viewer shows the video to 2 new people.

4. How many times was the video viewed on day n?

To model the sensation of "viral videos", assume that on day one there was one view, that every new view corresponds to a new person seeing the video and on average a new viewer shows the video to 2 new people.

4. Let v_1, v_2, v_3, \ldots be a sequence such that

 $v_n =$ (the number of times the video is viewed on the n^{th} day).

Write down an algebraic relationship between v_n and v_{n+1} .

To model the sensation of "viral videos", assume that on day one there was one view, that every new view corresponds to a new person seeing the video and on average a new viewer shows the video to 2 new people.

5. Let v_1, v_2, v_3, \ldots be a sequence such that

 $v_n =$ (the number of times the video is viewed on the n^{th} day).

Write down an algebraic relationship between v_n and v_{n+1} .

To model the sensation of "viral videos", assume that on day one there was one view, that every new view corresponds to a new person seeing the video and on average a new viewer shows the video to 2 new people.

- 6. How many times is the video viewed in the first two days?
- 7. How many times is the video viewed in the first three days?
- 8. How many times is the video viewed in the first five days?

To model the sensation of "viral videos", assume that on day one there was one view, that every new view corresponds to a new person seeing the video and on average a new viewer shows the video to 2 new people.

9. How many times is the video viewed in the first n days?

The graph below is data from YouTube about the actual number of views of Gangnam Style. Does our model accurately describe the behavior of the viral video phenomena? What do you think some limitations of our model are?



A ball is dropped from a height of 10 feet. The ball bounces to 80% of its previous height with each bounce.

- 1. How high does the ball bounce after the first bounce?
- 2. How high does the ball bounce after third bounce?

A ball is dropped from a height of 10 feet. The ball bounces to 80% of its previous height with each bounce.

- 3. How high does the ball bounce after the n^{th} bounce?
- 4. Let b_1, b_2, b_3, \ldots be a sequence where b_n is the height the ball bounces after the n^{th} bounce. What is the relationship between b_n and b_{n+1} ?

A ball is dropped from a height of 10 feet. The ball bounces to 80% of its previous height with each bounce.

5. Record (n, b_n) in a table and graph.

n	b_n
1	8
2	
3	
4	
5	
6	
10	



The sequence b_n models the height of a ball bouncing. How many times does the model predict the ball will bounce? Is this realistic?

Assume you invest \$1,000 in a savings account that pays 5% a year.

1. How much money will you have after one year?

2. How much money will you have after two years?

3. How much money will you have after 50 years?

Assume you invest 1,000 in a savings account that pays 5% a year.

4. How much money will you have after n years?

Assume you invest \$1,000 in a savings account that pays 5% a year.

5. Let m_1, m_2, m_3, \ldots be the sequence such that $m_n =$ dollars in account after *n* years. What is the relationship between m_n and m_{n+1} ?

The height of a ball bouncing, the number of viral video daily views, and the amount of money in the bank account are all examples geometric sequences. Describe similarities and differences among these three examples.

Counting High-Fives - Section 2.4 Proper Sports Etiquette

After a sporting event, the opposing teams often line up and exchange high-fives. Afterward, members of the same team exchange high-fives. In this problem, you will explore the total number of high-fives that take place at the end of a game.

- Every player exchanges exactly one high-five with every other player.
- ▶ When two players exchange a high-five, it counts as one exchange, not two.

Let n be the *combined* number of players on each of the two teams. Let H_n be the number of high-fives that are exchanged at the end of the game.

Complete the following table.

n players	H_n high-fives
1	
2	
3	
4	
5	
6	
7	

Counting High-Fives - Section 2.5 Question 2.14 - Click In!

Is the sequence H_1, H_2, H_3, \ldots an arithmetic sequence? Is the sequence geometric?

- 1. Arithmetic
- 2. Geometric
- 3. Neither

Sketch the graph of the high-five sequence, then describe what you see, comparing and contrasting it to the patio sequence from Question 2.3.



2.16 What is the relationship between H_n and H_{n-1}^{1} .

¹Think about the context of the problem. If the 4^{th} player enters the scene, how many players are there with whom she has to exchange high fives? What is the 5th player enters after her?

Find an explicit formula for the sequence H_n .

The defining quality of an arithmetic sequence is the constant difference between consecutive terms in the sequence.

1. Is there a constant difference between terms in the hand shake sequence?

The following picture describes how to find **the second difference** of a sequence:



2. Add 3 terms to the top row and complete the picture.

3. What is the pattern?

2. Calculate a few second differences of the hand shake sequence. To help you organize your thoughts, use the following diagram:



Make up your own sequence which has a constant second difference.

Make up your own sequence which has a constant third difference.

Make up your own sequence which has a constant third difference.

Counting High-Fives - Section 2.5 Polygon

A **polygon** is a closed shape consisting of line segments which pairwise share a common point. Below are drawn 3-sided, 4-sided, 5-sided and 6-sided polygons which you may know under different names.



Counting High-Fives - Section 2.5 Polygon

A **diagonal** of a polygon is a line segment which connects non-adjacent vertices of the polygon. Draw all diagonals for each polygon pictured above. Let's consider the sequence $\{d_n\}$ where d_n is the number of diagonals of a polygon with n sides.



1. Fill in the following table and sketch the graph for $\{d_n\}$.

n sides	d_n diagonals
1	
2	
3	
4	
5	
6	
7	



- 3. Can you come up with a recursive formula for the sequence?
- 4. Can you come up with an explicit formula for the sequence? 2

 $^{^2 {\}rm Once}$ again, it's useful to think about the context of the problem.

The sequence t_1, t_2, t_3, \ldots is given by the following table:

n	1	2	3	4	 n
t_n	2	5	8	11	 3n - 1

1. Graph the first few terms of t_n .



2. Does it make sense to connect the dots? Explain.



The sequence t_1, t_2, t_3, \ldots is given by the following table:

n	1	2	3	4	 n
t_n	2	5	8	11	 3n - 1

3. What is the relationship between t_n and t_{n+1} ?

4. On the same grid paper, make a graph for the sequence where the n^{th} term is $s_n = 3n + 1$. Compare your graph with the one you drew in the previous problem. How are they the same? How are they different?



Let's look at odd numbers! 1 is the first odd number, 3 is the second odd number, and so on. What is the 31^{st} odd number?

1. In terms of n what is the n^{th} odd number?
More Sequences - Section 2.6 Question 2.24 - Click In!

Does the sequence of odd numbers $1, 3, 5, 7, \ldots$ form a geometric sequence, arithmetic sequence, or neither? How do you know?

1. geometric

2. arithmetic

3. neither

4. Graph the first few terms of the sequence of odd numbers. Do the points you graphed lay on a straight line?



Look at the figure below and answer the questions that follow.



- 1. How many squares are in the top row?
- 2. How many squares are in the second row?
- 3. How many squares are in the fourth row?

Look at the figure below and answer the questions that follow.



3. If the figure were extended indefinitely forever, how many squares would be in the n^{th} row?



 $1, 3, 5, 7, \ldots$

4. Which question(s) that we have already answered does this relate to?



5. How many unit squares are in the first row? (a unit triangle is the smallest one in the picture)

- 6. How many unit squares are in the first two rows?
- 7. How many squares are in the first n rows?



5. How many squares are in the first n rows?

6. What is the sum of the first n odd numbers?

Suppose you put 2 cents in a jar today and each day thereafter you triple the amount you put in the previous day. How much would you put in on the 17th day? How big must your jar be? More Sequences - Section 2.6 Question 2.28 - Click In!

Is there a sequence that can be claimed to be both arithmetic and geometric? Explain.

- 1. Yes.
- 2. No.

> Miguel was asked to consider the pattern 0, 5, 10, 15, ... and list the next term. Miguel said 24. Can you figure out why Miguel chose that instead of 20?