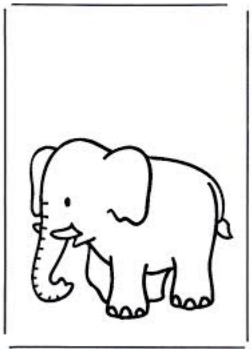
Quadratic Functions - Rectangular Fences Section 5.1

Question 5.1

You want to make a rectangular pen for Ellie, your pet elephant. What?! You don't have a pet elephant? That's rather unfortunate; they're quite cute. Well, imagine you have one. You want to make sure Ellie has as much space as possible. Unfortunately, you only have 28 feet of fencing available. If you use all of your fencing to make the pen, what is the biggest possible area?

Outline here possible approaches to answering this question. What might you, or someone else, try to do to solve this problem?



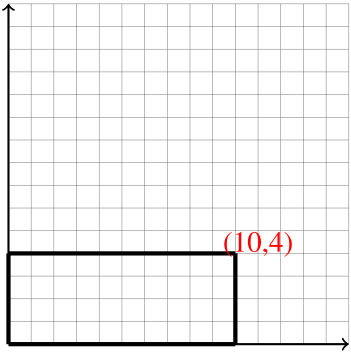
Quadratic Functions - Rectangular Fences Section 5.1

Question 5.2

We will investigate our main problem in several steps.

a. Draw 6 rectangular pens having a perimeter of 28 on the coordinate axis. Like below.

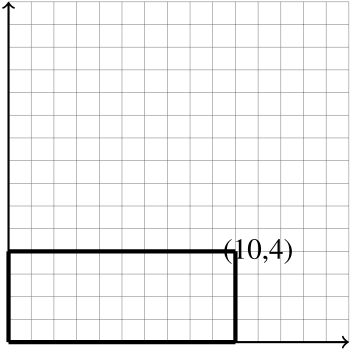
b. Label the coordinate in the upper right hand corner of each pen.



Quadratic Functions - Rectangular Fences Section 5.1

Question 5.2

c. Make a table showing all the coordinates on your graph. Look for a pattern and make three more entries in the table.



length	height
10	4

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.2

d. Write an equation for the function described by your graph and table. This is a function that will relate the height of the rectangle as a function of the length.

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.3

The point $(4, 10)$ is the upper right corner of a plausible pen.

a. What does the sum of these numbers represent in this problem?

b. What does the product of these two numbers represent in this problem?

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.3 - Click in!

c. Of all the rectangular pens recorded on your chart, which rectangular pen enclosed the largest area?

d. How many rectangles are there whose perimeter is 28? Click in!

I 28
II 29
III 56
IV More than 100.

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.4

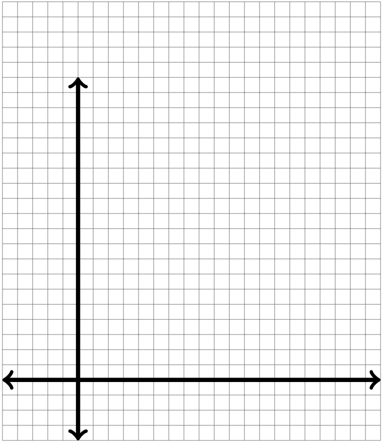
For each rectangle from **Question c.** compute the area.

length	height	area
10	4	

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.5

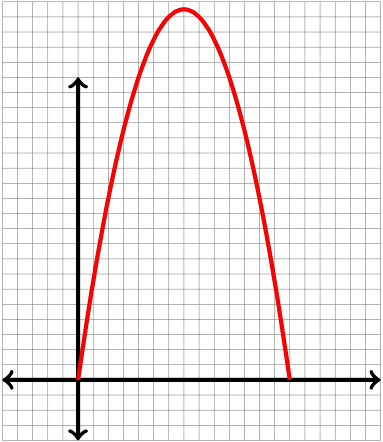
Make a graph of the area as a function of length. Connect the points on your graph with a smooth curve. What kind of curve is it?



Quadratic Functions - Rectangular Fences Section 5.1

Question 5.5

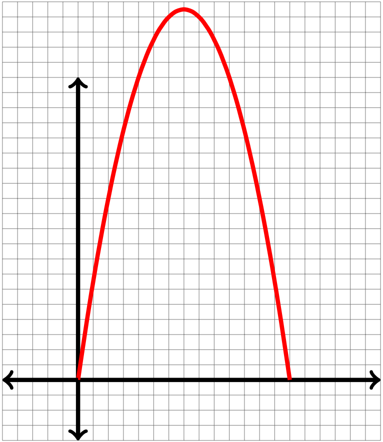
Make a graph of the area as a function of length. Connect the points on your graph with a smooth curve. What kind of curve is it?



Quadratic Functions - Rectangular Fences Section 5.1

Question 5.5

On the same coordinate system, make a graph of the area as a function of height. Connect the points on your graph with a smooth curve. What kind of curve is it? What else do you notice?



Quadratic Functions - Rectangular Fences Section 5.1

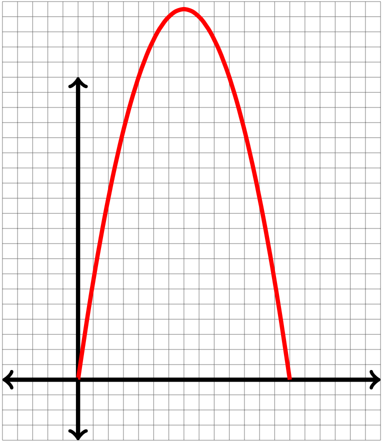
Question 5.7

Why did it make sense to connect the dots of both graphs?

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.8

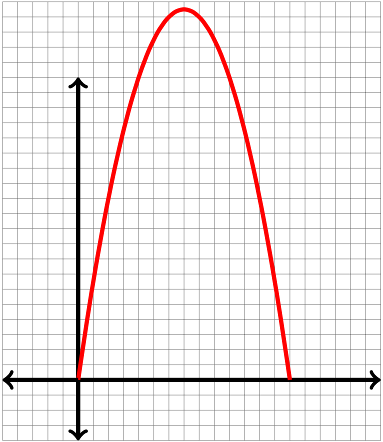
a. Label the highest point on your graph from 5.5 with its coordinates. Interpret these two numbers in terms of this problem. [1]



Quadratic Functions - Rectangular Fences Section 5.1

Question 5.8

b. Where does the graph cross the x-axis? What do these numbers mean?



Quadratic Functions - Rectangular Fences Section 5.1

Question 5.8

c If you increase the length by one foot, does the area increase or decrease? Does it change the same amount each time? Explain.

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.9

a. Describe in words how you would find the area of the rectangular pen having perimeter 28, if you knew its length.

b. If the perimeter of the rectangular pen is 28 and its length is L, write an algebraic expression for its area in terms of L.

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.9

c. If you had 28 feet of fencing and wanted to make the largest possible rectangular pen, what would its length, width, and area be? Explain.

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.10

a. Describe in words how you would find the area of the rectangular pen having perimeter P if you knew its length.

b. If the perimeter of the rectangular pen is P and its length is L, write an algebraic expression for its area in terms of L and P.

Quadratic Functions - Rectangular Fences Section 5.1

Question 5.10

c. If you had P feet of fencing and wanted to make the largest possible rectangular pen, what would its length, width, and area be? Explain.

Quadratic Functions - Graphs of quadratic functions 5.2

Question 5.11

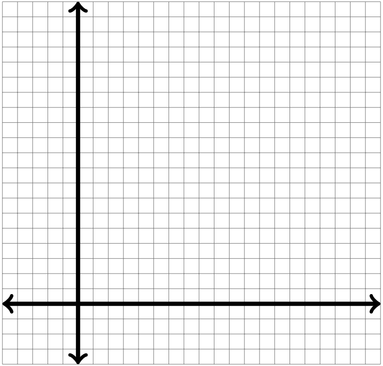
Graph each of the following functions. Use a scale that will show values from -5 to 20 for the domain and from -20 to 100 for the target. To graph the functions, make a table and plot points.

a. $f(x) = x(8-x)$

b. $g(x) = x(15-x)$

c. $h(x) = x(12-x)$

d. $k(x) = x(5-x)$



Quadratic Functions - Graphs of quadratic functions 5.2

Question 5.11

For $f(x) = x(8-x)$:

a. label the graph with its equation;

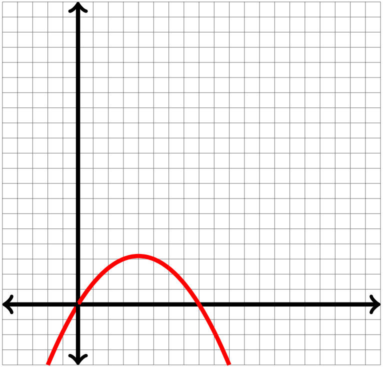
b. label the x-intercepts;

c. label the y-intercepts;

d. label the vertex;

e. draw the line of symmetry;

f. note if the graph opens up or opens down.



Quadratic Functions - Rectangular Fences Section 5.2

Question 5.11

For $g(x) = x(15 - x)$:

a. label the graph with its equation;

b. label the x-intercepts;

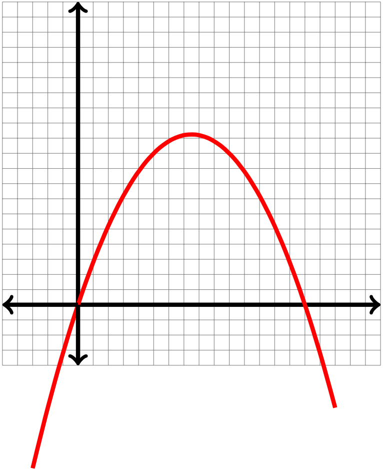
c. label the y-intercepts;

d. label the vertex;

e. draw the line of symmetry;

f. note if the graph opens up or opens down.

g. by looking at the graph note if any of the functions have an inverse function.



Quadratic Functions - Graphs of quadratic functions 5.2

Question 5.11

For $k(x) = x(5 - x)$:

a. label the graph with its equation;

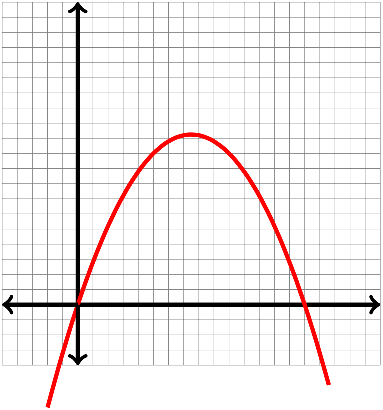
b. label the x-intercepts;

c. label the y-intercepts;

d. label the vertex;

e. draw the line of symmetry;

f. note if the graph opens up or opens down.



Quadratic Functions - Graphs of quadratic functions 5.2

Question 5.13

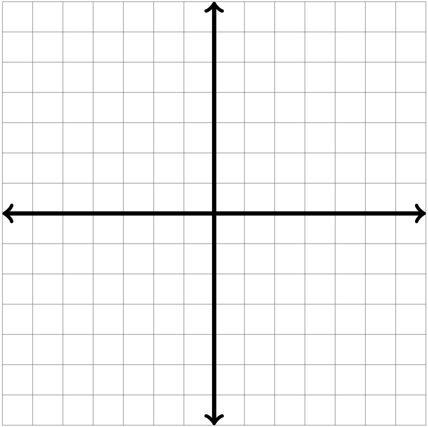
Describe the graph of the quadratic equation $f(x) = x(b - x)$. Write an expression for:

a. the x-intercepts:

b. the y-intercepts:

c. What is the line of symmetry for the graph?

d. What is the maximum value of f.



Quadratic Functions - Graphs of quadratic functions 5.2

Question 5.14

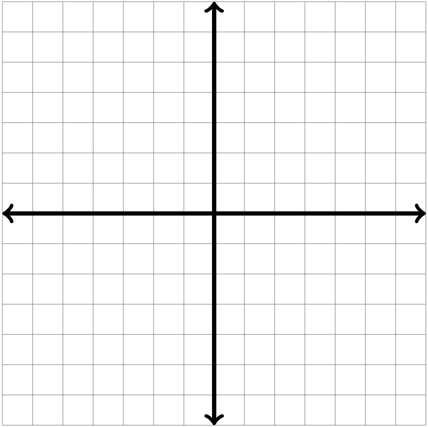
Describe the graph of the quadratic equation $f(x) = x(x - q)$. Write an expression for:

a. the x-intercepts:

b. the y-intercept:

c. What is the line of symmetry for the graph?

d. What is the minimum value of f.



Quadratic Functions - Graphs of quadratic functions 5.2

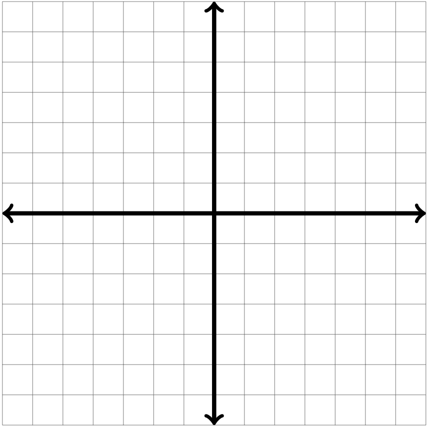
Question 5.15

Describe the graph of the quadratic equation $f(x) = (x-a)(x-b)$.
Write an expression for:

a. the x-intercepts:

b. the y-intercept:

c. What is the line of symmetry for the graph?



Sketch the graph!

Quadratic Functions - Graphs of quadratic functions 5.2

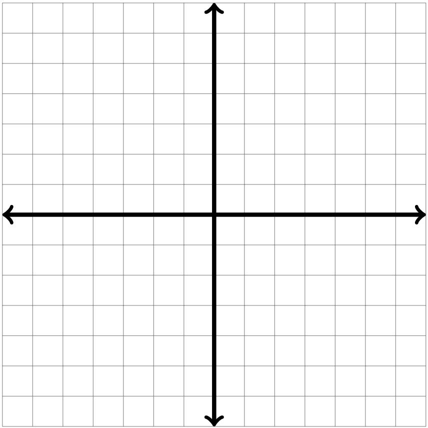
Question 5.16

Graph the function $f(x) = x^2 + x - 6$. Write an expression for

a. the x-intercepts;

b. the y-intercept.

c. What is the line of symmetry for the graph?



Sketch the graph!

Quadratic Functions - The zero product property 5.3

Question 5.17 - Click in!

If $ab = 0$, which of the following is impossible? Explain.

a. $a \neq 0$ and $b \neq 0$

b. $a \neq 0$ and $b = 0$

c. $a = 0$ and $b \neq 0$

d. $a = 0$ and $b = 0$

Proposition 1

Property

When the product of two quantities is zero, one of the quantities must be zero.

Quadratic Functions - The zero product property 5.3

Question 5.18

If $(x-6)(-2x-1)=0$, what are the possible values for x? (Hint: use Property 1)

Quadratic Functions - The zero product property 5.3

Question 5.19

What would Property 1 say if the product of three quantities equaled 0?

$$a \cdot b \cdot c = 0$$

Quadratic Functions - The zero product property 5.3

Question 5.19

Use Property 1 to solve the following equations:

a. $(3x+1)x = 0$

b. $(2x+3)(10-x) = 0$

c. $(3x-3)(4x+16) = 0$

d. $6x^2 = 12x$

Quadratic Functions - The zero product property 5.3

Factor

Definition

*An integer q is a **factor** of the integer p if there is a third integer g such that*

$$p = gq.$$

Quadratic Functions - Rectangular Fences Section 5.3

Factor

Definition

*A polynomial $q(x)$ is a **factor** of the polynomial $p(x)$ if there is a third polynomial $g(x)$ such that*

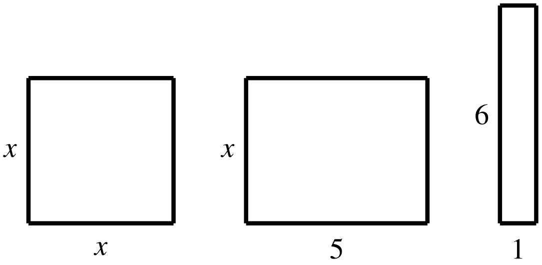
$$p(x) = q(x)g(x).$$

Quadratic Functions - The zero product property 5.3

Question 5.21

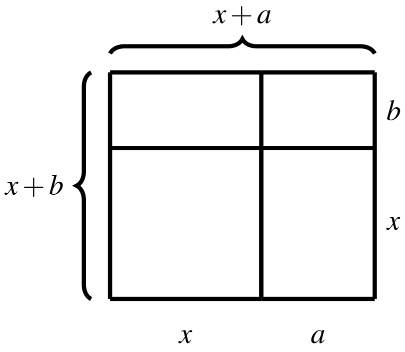
Solve $x^2+5x+6=0$. Our goal is to accomplish this by writing the left hand side as a product of two linear expressions, and then using the zero product property to find the solutions.

a. Each term on the left hand side of the equation has a geometric meaning:



Question 5.21

b. When we factor x^2+5x+6 we are representing the above area as the area of a rectangle.



c. Find a and b such that $x^2+5x+6=(x+a)(x+b)$. Use the picture.

Quadratic Functions - The zero product property 5.3

Question 5.21

d. Now that we have factored $x^2+5x+6=(x+3)(x+2)$, solve the equation $x^2+5x+6=0$

Quadratic Functions - The zero product property 5.3

Question 5.22

Solve the following equations by factoring. To help you factor draw the picture from Question **??**.

a. $x^2 + 6x + 9 = 0$

b. $x^2 + 12x + 35 = 0$

c. $x^2 + 9x + 20 = 0$

Quadratic Functions - The zero product property 5.3

Question **??** - Click in!

Not every quadratic polynomial can be factored. Which one of following polynomial functions can not be factored? You will want to graph each of them. Filling out a table should help.

Click in!

a. $f(x) = x^2 + 10x + 25$

b. $g(x) = x^2 + 7x + 5$

c. $h(x) = x^2 + 10x + 21$

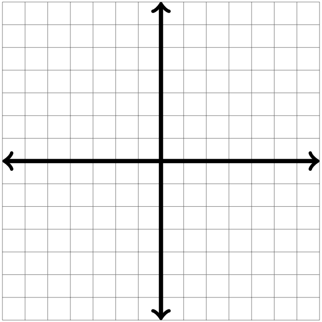
d. $k(x) = x^2 - 6x + 10$

Quadratic Functions - The zero product property 5.3

Question ??

a. $f(x) = x^2 + 10x + 25$

x	$f(x)$

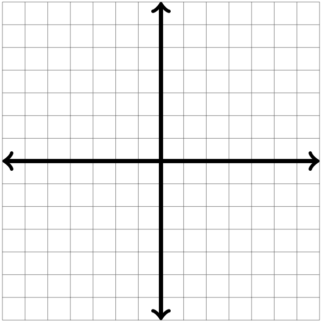


Quadratic Functions - The zero product property 5.3

Question ??

a. $g(x) = x^2 + 7x + 5$

x	$g(x)$

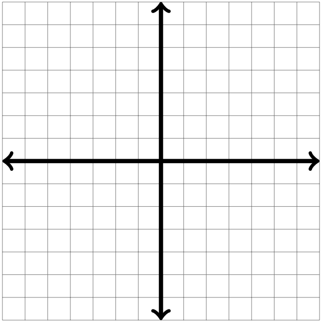


Quadratic Functions - The zero product property 5.3

Question ??

a. $h(x) = x^2 + 10x + 21$

x	$h(x)$

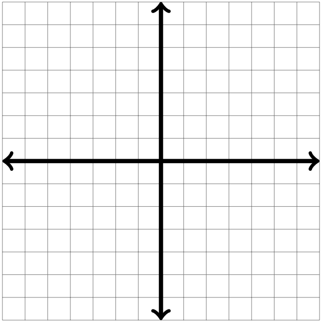


Quadratic Functions - The zero product property 5.3

Question ??

a. $k(x) = x^2 - 6x + 10$

x	$k(x)$



Quadratic Functions - Completing the Square 5.4

Question 5.25

Solve the following equations (hint each equation has two solutions):

a. $x^2 = 4$

b. $x^2 = 25$

c. $x^2 = 7$

d. $(x+3)^2 = 16$

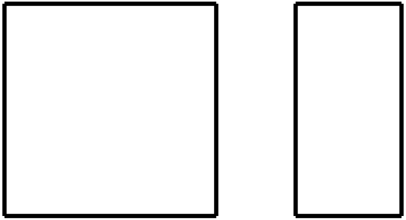
e. $(x+4)^2 = 5$

Quadratic Functions - Completing the Square 5.4

Question 5.26

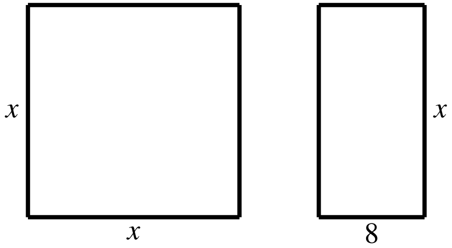
In Question 5.25 we were able to solve the equation $x^2+8x+11=0$. Try and factor $x^2+8x+11$. In this question we are going to investigate how to turn $x^2+8x+11=0$ into the more convenient form of $(x+4)^2=5$.

a. Label the sides of the square and rectangle below so that the total area is x^2+8x.



Quadratic Functions - Completing the Square 5.4

Question 5.26



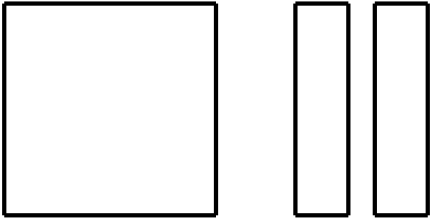
b. Our goal is to cut and rearrange the pieces we have so that the new shape resembles a square as much as possible. What would you do?

c. Why do we want a square?

Quadratic Functions - Completing the Square 5.4

Question 5.26

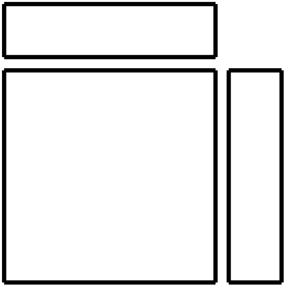
d. Here is how one student did this: she chopped the $8x$ rectangle in half. Label each side length. Has the area changed?



Quadratic Functions - Completing the Square 5.4

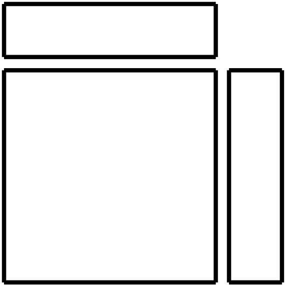
Question 5.26

e. In the picture below one of the rectangles has been moved to the top. Label the side lengths. Has the area changed?



Quadratic Functions - Completing the Square 5.4

Question 5.26



f. Notice that this arrangement almost makes a square. What would be the area of the entire square?

g. What is the area of the missing piece?

Quadratic Functions - Completing the Square 5.4

Question 5.26

h. Write an algebraic equation that relates: the area of the entire square, the area of missing piece, and x^2+8x.

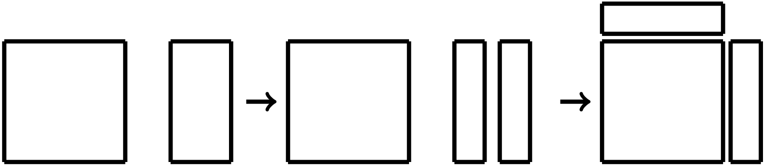
i. Use Part h. to substitute for x^2+8x in the equation $x^2+8x+11=0$.

Quadratic Functions - Completing the Square 5.4

Question 5.27

The process that we carried out in Question 5.26 is called completing the square. It is **wonderfully useful**. Complete the square for the following expression. Use the pictures provided to organize your thoughts.

a. $x^2 + 10x$

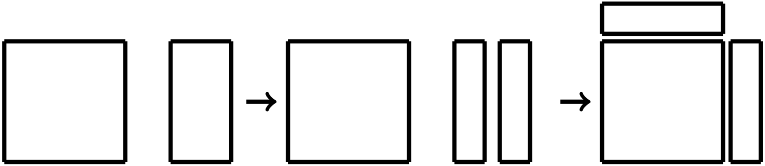


Quadratic Functions - Completing the Square 5.4

Question 5.27

The process that we carried out in Question 5.26 is called completing the square. It is **wonderfully useful**. Complete the square for the following expression. Use the pictures provided to organize your thoughts.

b. $x^2 + 12x$

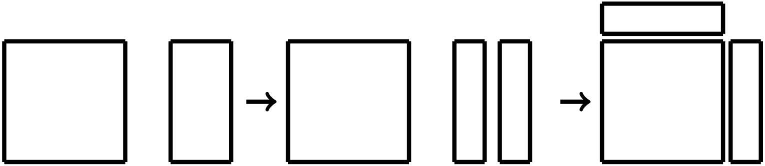


Quadratic Functions - Completing the Square 5.4

Question 5.27

The process that we carried out in Question 5.26 is called completing the square. It is **wonderfully useful**. Complete the square for the following expression. Use the pictures provided to organize your thoughts.

c. $x^2 + 5x$

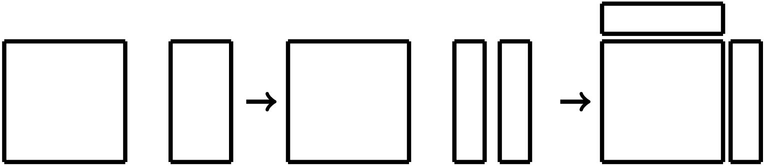


Quadratic Functions - Completing the Square 5.4

Question 5.27

The process that we carried out in Question 5.26 is called completing the square. It is **wonderfully useful**. Complete the square for the following expression. Use the pictures provided to organize your thoughts.

b. $x^2 + 12x$



Quadratic Functions - Completing the Square 5.4

Question 5.28

In Question 5.27 we completed the square for several expressions. Use that information to solve the following equations:

a. $x^2 + 10x = 10$

b. $x^2 + 12x = 14$

c. $x^2 + 5x = 7$

Quadratic Functions - Completing the Square 5.4

Question 5.29

Let's take this up a notch. Solve the following equations:

a. $2x^2 - 4x - 16 = 0$

b. $2x^2 + x - 6 = 0$

Quadratic Functions - Calculating Max and Min Values 5.5

Question 5.30

David Ortiz of the Boston Red Sox has an average off the bat speed of 102.2 miles per hour in the 2013 play off season. The average vertical speed off the bat is 67.5 miles per hour. This means that the height of the ball is given by $h(t) = -21.9t^2 + 67.5t$.

a. How long is the ball in the air?

b. What is the maximum height of the ball?

Quadratic Functions - Calculating Max and Min Values 5.5

Question ??

Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2 + 3x - 4$. Does f have a maximum or a minimum. How were you able to tell?

Quadratic Functions - Calculating Max and Min Values 5.5

Question 5.32

The following questions lead us to discover what minimum value of $f(x) = x^2 + 3x - 4$ is.

a. Given that $f(x) = x^2 + 3x - 4$ what are the values of x such that $f(x) = 0$?

b. Use the symmetry of the graph of f to calculate the value of x where f achieves its minimum or maximum.

Quadratic Functions - Calculating Max and Min Values 5.5

Question 5.32

The following questions lead us to discover what minimum value of $f(x) = x^2 + 3x - 4$ is.

c. Use Part b. to find the minimum or maximum value of f.

d. Write $f(x)$ in a completed square form.

Quadratic Functions - Calculating Max and Min Values 5.5

Question 5.32

Sketch the graph of $f(x) = x^2 + 3x - 4$.