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Homework 8

Instructions: Please answer the following questions with well thought out answers. You can use this sheet to write your answers or use your own. STAPLE this sheet to the front of the rest of your work. It is important to be able to explain ideas clearly. In this assignment pretend that you are trying to explain to a friend your answers. You should strive for your answers to be: precise, accurate, succinct, and understandable.

Space Debris.

One problem that NASA and space scientists from other countries must deal with is the accumulation of space debris in orbit around Earth. Such debris includes payloads that are no longer operating; spent stages of rockets, assorted parts and lost tools; debris from the breakup of larger objects or from collisions between objects; and countless small pieces, such as flakes of paint and even smaller objects. Because bodies in Earth orbit travel at approximately 17500 miles per hour, a collision with even a tiny object can have catastrophic effects. In 1990, scientists estimated that a total of 4 million pounds of debris was in Earth orbit. They also estimated that at that time, we were adding 1.8 million pounds per year to the already serious problem, which in a few years would result in 9.5 million pounds of orbital debris. The 1990 prediction also stated that the amount of debris being added was anticipated to increase to a rate of 2.7 million pounds per year by the year 2000.

Question 1 How much is 4 million pounds of anything? Give three concrete examples of something weighing 4 millions pounds (example: 12549020 baseballs would weigh 4 million pounds).

1. First example:

2. Second example:

3. Third example:

Modeling the problem: Linear Growth

The problem of determining the amount of debris in space and the anticipated rate of increase of such matter is not one that can be solved directly. We cannot locate, count, and weigh all objects in orbit. Nor can we predict with assurance when two of them will collide. Instead, we must rely on mathematical models to help us represent the problems and identify trends and expected outcomes. In these activities, you will create and compare various mathematical models to help you investigate some of the questions raised by the proliferation of orbital debris. These models are greatly simplified in their assumptions so that you can investigate them with calculators, spreadsheets, and graphing utilities, but they provide insight into the process of mathematical modeling and its importance.

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Question 2 When creating models, mathematicians favor the simplest model that will account for the phenomena in question. Generally, a linear model gives the simplest case. So, using the reported 1990 rate of increase of 1.8 million pounds per year and assuming 4 million pounds of existing debris at the beginning of 1990, write a linear model to predict the number of pounds of orbital debris at the end of any given year, t. Assume that t = 1 represents 1990.

1. Linear model using rate of increase of 1.8 million pounds per year:

Question 3 Write a second linear model using the predicted 2.7 million pounds per year rate of increase and the initial 4 million pounds for 1909.

1. Linear model using rate of increase of 2.7 million pounds per year:

Question 4 Evaluate each model for several years to determine the year in which the predicted 9.5 million pounds of accumulated debris would occur.

1. What year does the first model predict there will be 9.5 million pounds of space debris?

2. What year does the second model predict there will 9.5 million pounds of space debris?

Question 5 Do you think that either of your linear models accurately represents that situation of escalating amounts of space debris as described in the original paragraph? Why or why not?

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Refining the Model: Quadratic Growth

Does either rate, 1.8 million pounds per year or 2.7 million pounds per year, tell us how much debris is building up between 1990 and 2000? Which rate of increase should we use? Obviously the amount being added each year is changing during this period, but by how much each year? The problem is one of acceleration, not constant velocity, so we need to adjust our model.

Again, let's make the simplest assumption: the rate at which we are adding debris increases at a constant rate from 1.8 million pounds per year in 1990 to 2.7 million pounds per year in 2000. This change means that over the ten-year period from the end of 1990 through 2000, the rate (velocity) of littering will increase by 0.9 million pounds (2.7 - 1.8 = 0.9), and we are making the assumption that this increase is achieved in equal annual increments of 0.09 million pounds per year in each year of the decade.

Question 6 Create a table to show the amount of debris added each year and the total amount in orbit at the end of the year.

Year	Amount added in year (millions of pounds)	Total in orbit at end of year (millions of pounds)
1990	1.8	5.8
1991		
1992		
1993		
1994		
1995		
1996		
1997		
1998		
1999		
2000		

Question 7 Since we assumed that the increase in the velocity of littering was achieved in equal annual increments, you can write a linear equation that describes the increase in the amount of debris being added each year (i.e, the increase in the annual velocity of littering as a function of the number of years since 1990. In this case, we let $a = 0$ in 1990 because we are assuming that the 1990 rate of 1.8 million pounds per year is our baseline rate. Then $d = f(t)$ represents the rate of littering t years after 1990.

1. Write a linear equation that describes the increase in the amount of debris being added each year:

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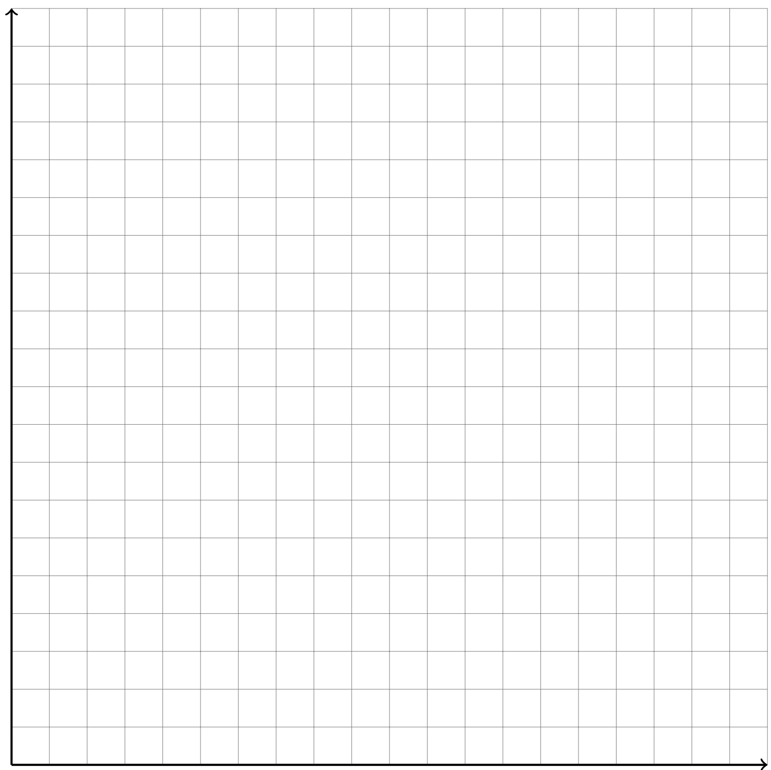
Question 8 The situation described in your equation, where the rate of increase of litter is itself increasing at a constant rate, is analogous to a vehicle that accelerates at a constant rate from an initial velocity, v_0, to a final velocity, v_f. Use the data generated in the table above to create a scatterplot of the total number of pounds of orbital debris that have accumulated relative to the year[1]. You graph should cover the period from 1990 through 2000.



Question 9 Fit a line to your data and decide whether the accumulation of debris appears to be linear. Write your conclusion and describe the evidence on which you based your decision.

1. Do you think the accumulation of the debris appears to be linear?

2. What evidence helped you reach this conclusion?

Question 10 Draw a line that approximates the data. Calculate the equation for that line.

1. What is the equation for your "best fit" line?

[1] In essence, graph the data from the table

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Question 11 If you asked a computer to give you a quadratic function that approximated your data, it would give you the following function $g(x) = 4 + 1.755x + .045x^2$. Graph this function on top of the scatter plot you created (use a different color if possible).

Question 12 Compare how $g(x) = 4 + 1.755x + .045x^2$ with the earlier linear models.

Question 13 In each case, use your models to predict the accumulation of debris after twenty years, thirty years, and fifty years. Describe the behavior of the linear model versus the quadratic model over time.

Year	Linear 1	Linear 2	Quadratic
1990			
2010			
2020			
2030			
2040			

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Refining the Model: Exponential Growth

Question 14 Instead of assuming that the acceleration of space garbage is constant, we could assume that amount of space garbage is constantly growing by 20%. Under this assumption we would have an exponential model

$$f(t) = 4e^{.2t}$$

Make a table for $f(t)$.

t	$f(t)$
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Question 15 How does the exponential function f compare to the data given in the problem?

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What Goes Up Might Come Down: Extending the Models

All the models you have developed thus far assume that the additional every year some of the debris slows down enough to re-enter the atmosphere where it burns up or, on rare occasions, returns to Earth. Assume for the moment that 10 percent of the debris in orbit at the beginning of any year will be destroyed during that year.

Question 16 Modify you linear, quadratic, and exponential models to account for the situation in which additional debris is being added each year while 10 percent of what was already in orbit is being destroyed. Here we won't be able to write down an explicit formula.

First Linear Model

1. Modify your first linear model and write debris as a sequence d_n
2. Write down a recursive relationship between step n and step $n+1$.
3. Make a table for the first 10 years using these sequences.

n	d_n
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Second Linear Model

1. Modify your second linear model and write debris as a sequence g_n.
2. Write down a recursive relationship between step n and step $n+1$.
3. Make a table for the first 10 years using these sequences.

n	g_n
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

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Quadratic Model

1. Modify your quadratic model and write debris as a sequence h_n

2. Write down a recursive relationship between step n and step $n+1$.

3. Make a table for the first 10 years using these sequences.

n	h_n
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Exponential Model

1. Modify your first exponential mode (this will require seeing it as a geometric sequence) and write debris as a sequence k_n.

2. Write down a recursive relationship between step n and step $n+1$.

3. Make a table for the first 10 years using these sequences.

n	k_n
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Question 17 In which case - linear, quadratic, or exponential - does the assumption of a 10% reentry rate have the greatest effect?

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Putting Your Models to Work

The power of mathematical models comes from their ability to enable us to ask ”What if?” questions. An earlier question is an example: What if we could increase the rate at which orbital debris is destroyed? Other questions might include these: What if we decrease the rate at which we are adding debris and find a way to increase the rate at which existing debris is destroyed? Such questions lead to open-ended investigations using mathematical models.

Question 18 Working with a partner or a small group, generate two or three specific questions that you would like to investigate.

1. What if ....
2. What if ...
3. What if ...

Question 19 Describe a plan for investigating your questions using spreadsheets, graphing utilities, or other appropriate technology.