Name: Number:

# Homework 2 - Power Functions
# Due: February 21

**Instructions:** Please answer the following questions with well thought out answers. Write your answers on this paper. It is important to be able to explain ideas clearly. In this assignment pretend that you are trying to explain to a friend your answers. You should strive for your answers to be: precise, accurate, succinct, and understandable.

**Definition 1** *A **power function** has domain $\mathbb{R}$ and target $\mathbb{R}$ and is defined by $x \mapsto cx^r$, where $c$ and $r$ can be any fixed real numbers.*

## Graphing by hand.

In the first half of the assignment we will focus on: $f(x) = x$, $g(x) = x^2$, $h(x) = x^3$, $j(x) = x^4$, $k(x) = x^5$, and $l(x) = x^6$. We will construct a table and use the table to sketch the graph of these functions.

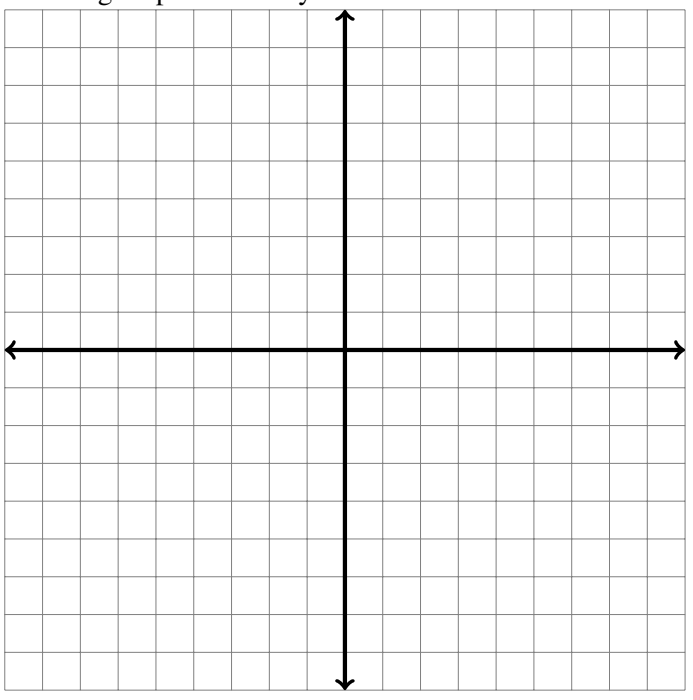
1. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x$.

   (a) The function $f(x) = x$ is also known as the "identity function." Why do you think it is called that?

   (b) Complete this table:

| $x$ | $f(x)$ |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| $-\frac{1}{2}$ | |
| 0 | |
| $\frac{1}{2}$ | |
| 1 | |
| 2 | |
| 3 | |

   (c) Carefully sketch the graph of $f$, remembering that the domain is $\mathbb{R}$, and noting the scale, and including all points from your table.



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(d) Write down two characteristics of this function that you notice, and commit them to memory.

(e) On the same pair of axes above, sketch the graph of the **inverse relation** of $f$. Is the inverse relation also a function?

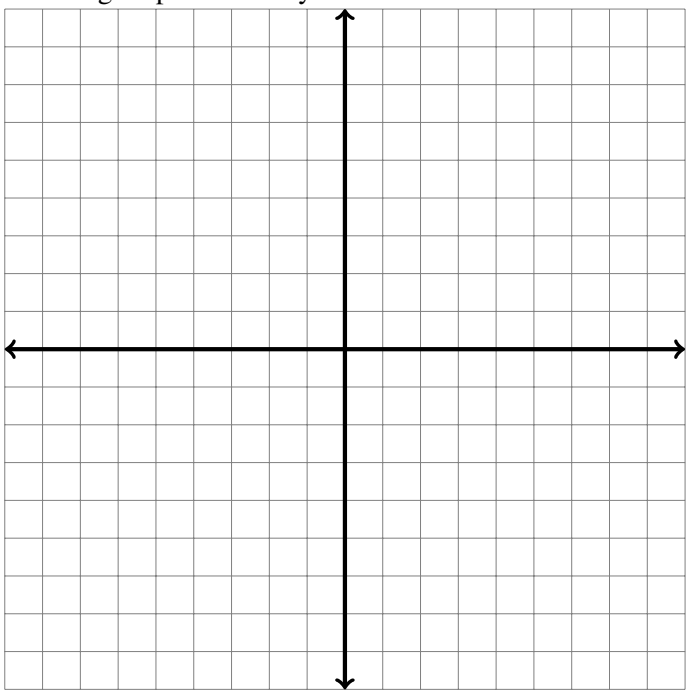
(f) Write down one characteristic of the inverse that you notice.

2. Consider the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^2$.

(a) Complete this table:

| $x$ | $g(x)$ |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| $-\frac{1}{2}$ | |
| 0 | |
| $\frac{1}{2}$ | |
| 1 | |
| 2 | |
| 3 | |

(b) Carefully sketch the graph of $f$, remembering that the domain is $\mathbb{R}$, and noting the scale, and including all points from your table.



(c) Write down two characteristics of this function that you notice, and commit them to memory.

(d) On the same pair of axes above, sketch the graph of the **inverse relation** of $f$. Is the inverse relation also a function?

(e) Write down one characteristic of the inverse that you notice.

(f) How is this graph similar and different from the previous function's graph?

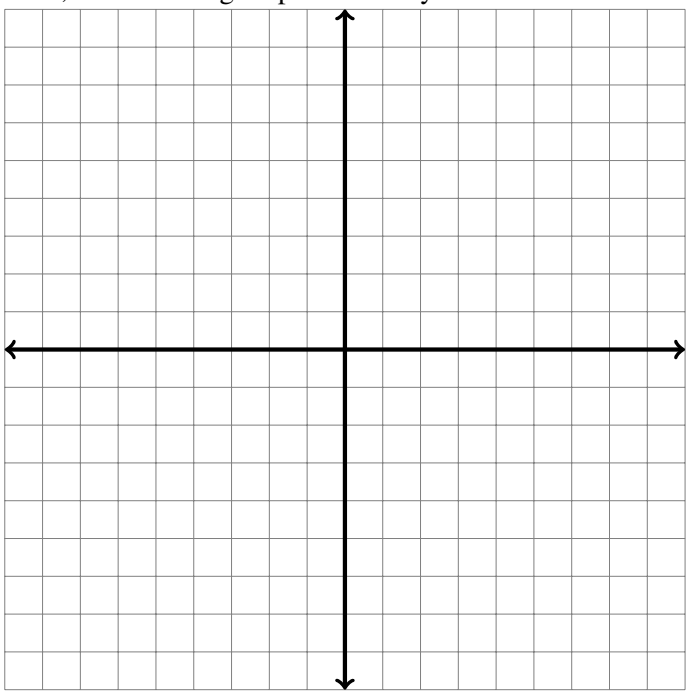
Name: Number:

3. Consider the function $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = x^3$.

(a) Complete this table:

| $x$ | $h(x)$ |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| $-\frac{1}{2}$ | |
| 0 | |
| $\frac{1}{2}$ | |
| 1 | |
| 2 | |
| 3 | |

Then carefully sketch the graph of $h$, remembering that the domain is $\mathbb{R}$, noting the scale, and including all points from your table.



(b) Write down two characteristics of this function that you notice, and commit them to memory.

(c) On the same pair of axes above, sketch the graph of the **inverse relation** of $h$. Is the inverse relation also a function? Write down one characteristic of the inverse that you notice.

(d) How is this graph similar and different from the previous function's graph?

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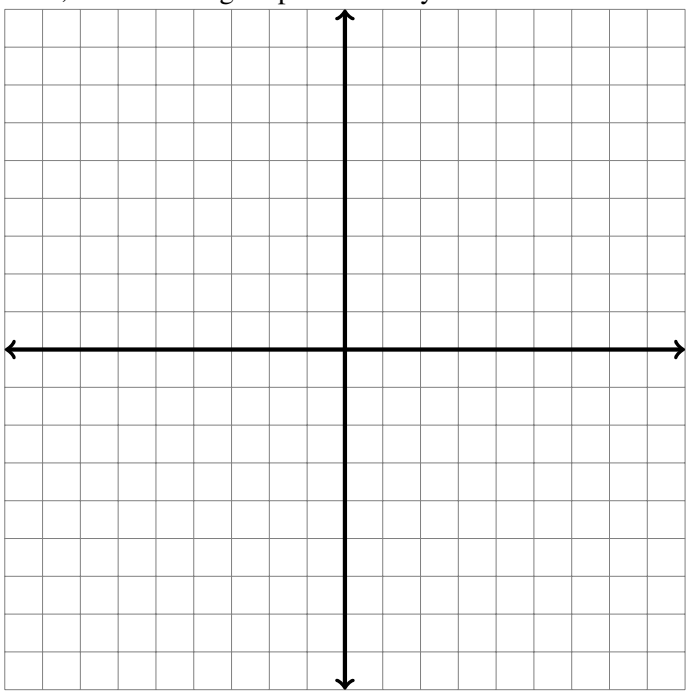
4. Consider the function $j : \mathbb{R} \to \mathbb{R}$ defined by $j(x) = x^4$.

(a) Complete this table:

| $x$ | $j(x)$ |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| $-\frac{1}{2}$ | |
| 0 | |
| $\frac{1}{2}$ | |
| 1 | |
| 2 | |
| 3 | |

Then carefully sketch the graph of $j$, remembering that the domain is $\mathbb{R}$, noting the scale, and including all points from your table.



(b) Write down two characteristics of this function that you notice, and commit them to memory.

(c) On the same pair of axes above, sketch the graph of the **inverse relation** of $j$. Is the inverse relation also a function? Write down one characteristic of the inverse that you notice.

(d) How is this graph similar and different from the previous function's graph?

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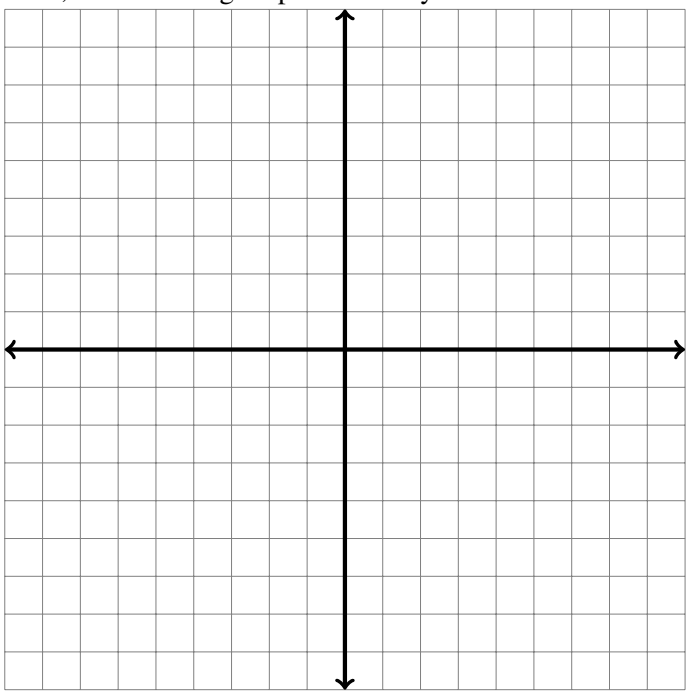
5. Consider the function $k : \mathbb{R} \to \mathbb{R}$ defined by $k(x) = x^5$.

(a) Complete this table:

| $x$ | $k(x)$ |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| $-\frac{1}{2}$ | |
| 0 | |
| $\frac{1}{2}$ | |
| 1 | |
| 2 | |
| 3 | |

Then carefully sketch the graph of $k$, remembering that the domain is $\mathbb{R}$, noting the scale, and including all points from your table.



(b) Write down two characteristics of this function that you notice, and commit them to memory.

(c) On the same pair of axes above, sketch the graph of the **inverse relation** of $k$. Is the inverse relation also a function? Write down one characteristic of the inverse that you notice.

(d) How is this graph similar and different from the previous function's graph?

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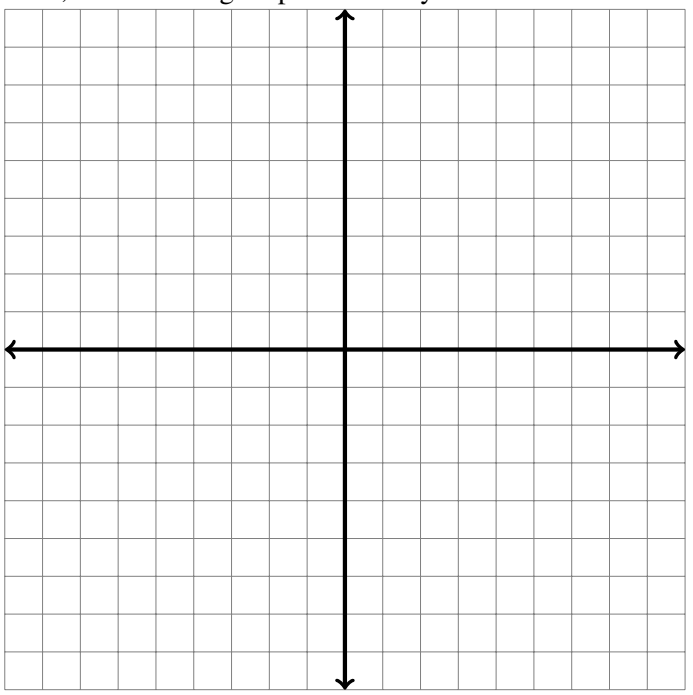
6. Consider the function $l : \mathbb{R} \to \mathbb{R}$ defined by $l(x) = x^6$.

(a) Complete this table:

| $x$ | $l(x)$ |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| $-\frac{1}{2}$ | |
| 0 | |
| $\frac{1}{2}$ | |
| 1 | |
| 2 | |
| 3 | |

Then carefully sketch the graph of $l$, remembering that the domain is $\mathbb{R}$, noting the scale, and including all points from your table.



(b) Write down two characteristics of this function that you notice, and commit them to memory.

(c) On the same pair of axes above, sketch the graph of the **inverse relation** of $l$. Is its inverse also a function? Write down one characteristic of the inverse that you notice.

(d) How is this graph similar and different from the previous function's graph?

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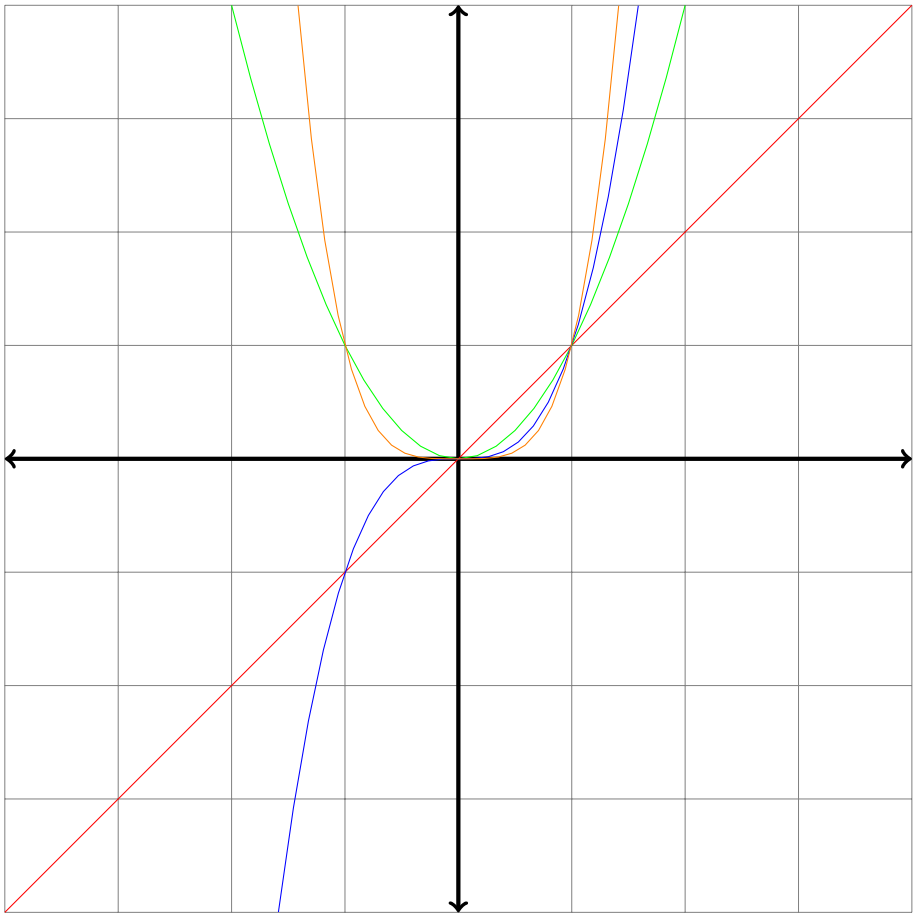
7. Describe what you think the graph of $m : \mathbb{R} \to \mathbb{R}$ defined by $m(x) = x^7$ looks like? Why do you think that? What about the power function $n(x) = x^8$? Why?

8. Label each of the power functions graphed below with one of the following:
$f(x) = x, g(x) = x^2, h(x) = x^3, j(x) = x^4$.



9. What points do the functions have in common?

10. For $0 < x < 1$, write $f(x), g(x), h(x)$, and $j(x)$ in ascending order.

11. For $x > 1$, write $f(x), g(x), h(x)$, and $j(x)$ in ascending order.

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12. When does a power function have an inverse relation that is also a function?

13. Describe any symmetry you see in the "even" power functions.

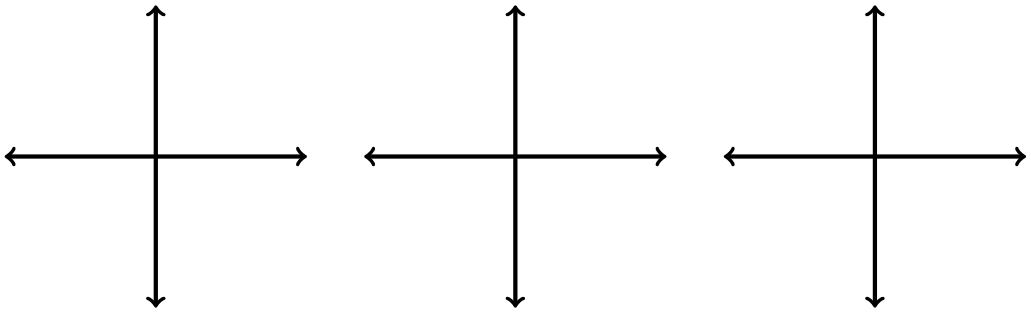
14. Describe any symmetry you see in the "odd" power functions.

## Graphing using technology.

In this section of the assignment you should make use of technology to help you graph the various functions. You can use a graphing calculator, or make use of one of the following the websites:

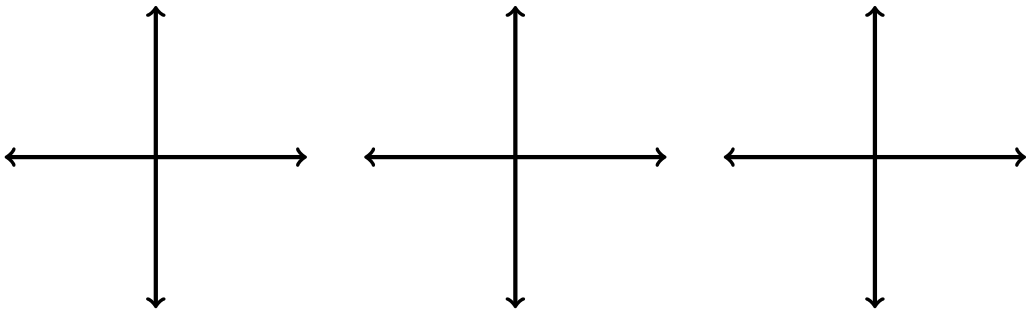
`http://www.wolframalpha.com`
`http://www.geogebra.org/cms/en/`

1. Use technology to sketch the graph of each of the following functions related to $f(x) = x^2$:



$f(x) = x^2$ $g(x) = (x+2)^2$ $g(x) = (x-2)^2$

2. Use technology to sketch the graph of each of the following functions related to $h(x) = x^3$:



$f(x) = x^3$ $g(x) = (x+2)^3$ $h(x) = (x-2)^3$

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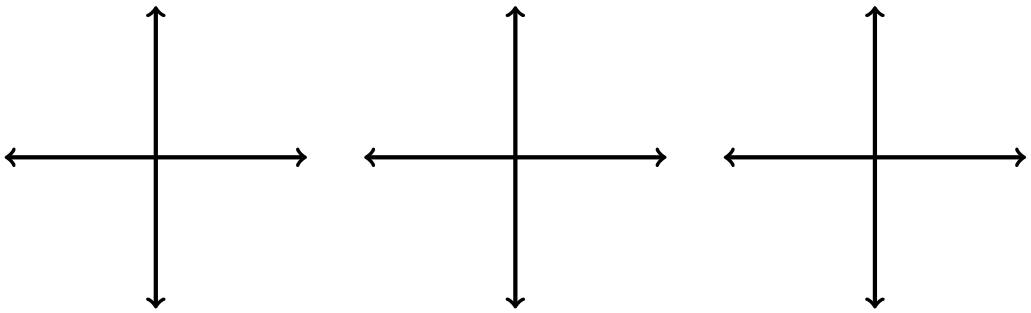
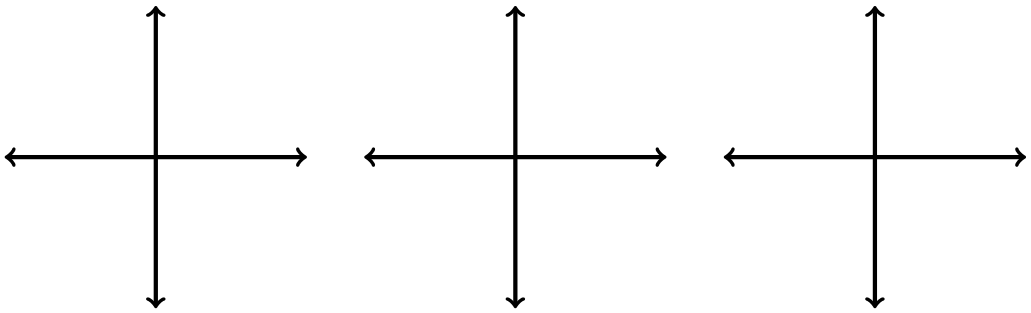
3. Use Question 1 and Question 2 to make a conjecture about how the graphs of $f(x) = x^4, g(x) = (x+2)^4$ and $h(x) = (x-2)^4$ compare.

4. Decide if your conjecture is true by using technology to graph each of the following:



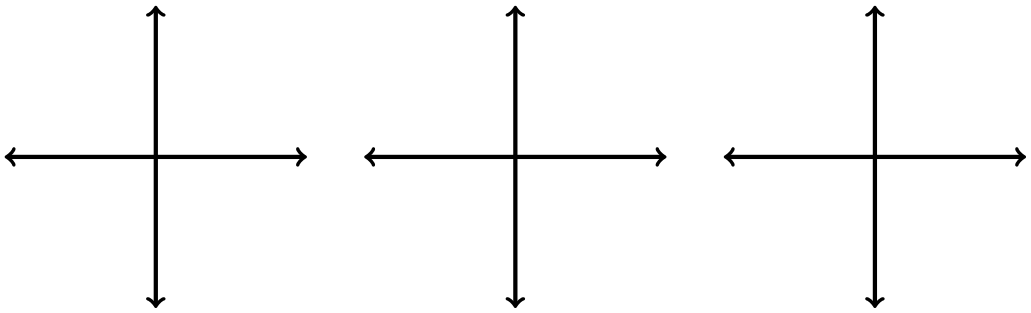
$f(x) = x^4$ $g(x) = (x+2)^4$ $h(x) = (x-2)^4$

5. Use technology to sketch the graph of each of the following functions related to $f(x) = x^2$:



$f(x) = x^2$ $g(x) = x^2 + 2$ $g(x) = x^2 - 2$

6. Use technology to sketch the graph of each of the following functions related to $h(x) = x^3$:



$f(x) = x^3$ $g(x) = x^3 + 2$ $h(x) = x^3 - 2$

7. Use Question 5 and Question 7 to make a conjecture about how the graphs of $f(x) = x^4, g(x) = x^4 + 2$ and $h(x) = x^4 - 2$ compare.

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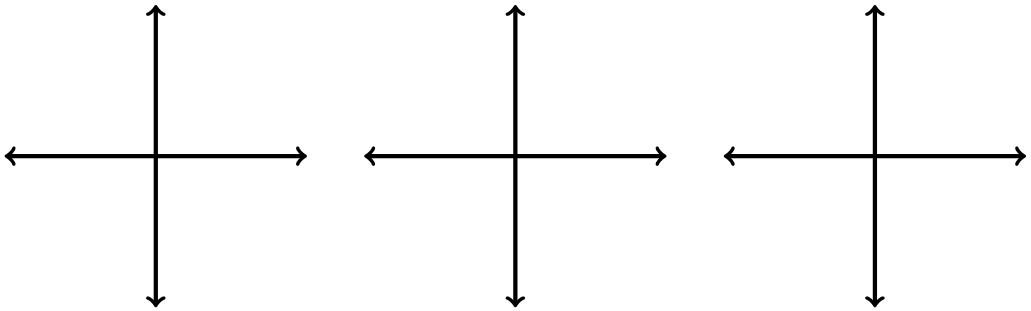
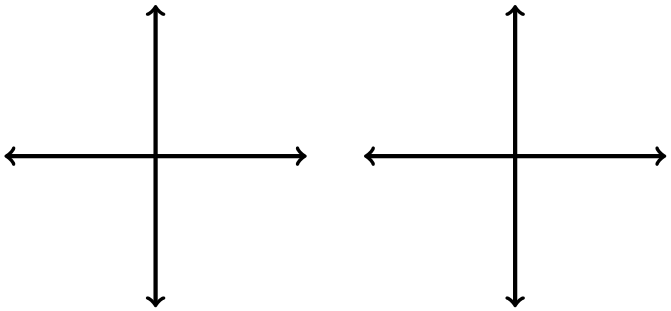
8. Decide if your conjecture is true by using technology to graph each of the following:



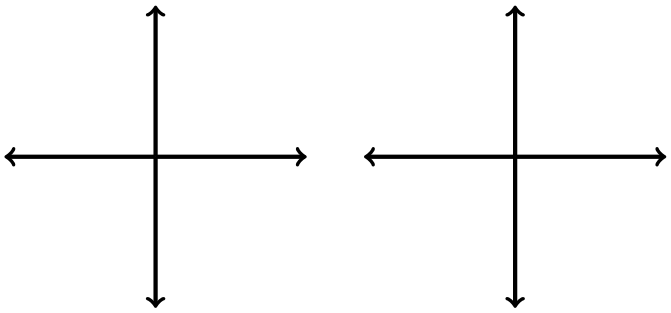
$f(x) = x^4$ $g(x) = x^4 + 2$ $h(x) = x^4 - 2$

9. Given a function $f(x)$ make a conjecture about how the graphs for $f(x)$ and $-f(x)$ are related (hint: think about how the outputs are different).

10. Test your conjecture Question 9 by using technology to graph the following graphs:



$f(x) = x^4$ $-f(x) = -x^4$



$f(x) = x^3$ $-f(x) = -x^3$