Homework 1 - College Savings Plan Due: February 10

Instructions: Please answer the following questions with well thought out answers. Write your answers on a separate sheet of paper. STAPLE this sheet to the front of your answers. It is important to be able to explain ideas clearly. In this assignment pretend that you are trying to explain to a friend your answers. You should strive for your answers to be: precise, accurate, succinct, and understandable. In this activity you will mathematically describe a college savings plan. We begin by developing some Excel skills.

Recursive Relationships in Excel

Definition 1 A cell is one entry in the Excel spreadsheet.

- 1. Find the Excel document on my website entitled CollegeSavingsPlan.xlsx and save it as CollegeSavingsPlan.yourlastname.yourfirstname.xlsx
- 2. The table Recursive Relationships 1 shows a geometric sequence.
 - What is the relationship between a_n and a_{n+1} ?
 - What is the first term?
 - Write an explicit formula for a_n .
 - In the table we see that the second term is 15. Click on the 15 and look at how that cell is "computed". Describe what you see.
- 3. The table Recursive Relationships 2 shows the beginning of an arithmetic sequence.
 - What is the relationship between a_n and a_{n+1} ?
 - What is the first term?
 - Write an explicit formula for a_n .
 - In the table we see that the second term is 14. Click on the cell that has the 14 in it. You will see that this cell is computed by = L9+2. This means is that this cell contains the value of the cell L9 plus 2.
 - After you click on the cell that has the 14, the border of the cell will become highlighted and there will be a small square in the lower right hand corner. Click and drag the small square. As you drag the square you will be selecting are region of cells. Drag the square and select the next five cells. Each cell in that region is now two more than the one above it.
- 4. The table Recursive Relationships 3 shows the first term of a sequence. In the sequence, the following relationship holds: $a_{n+1} = 4 \cdot a_n + 2$.
 - Is the sequence arithmetic or geometric?
 - Make use of what we learned in Part 3 to complete the table (write a formula and drag it to the subsequent cells).

Basic Interest

- 1. Record your answers from the parts below in the table labeled **Basic Savings Plan**. In this basic plan you will deposit a sum only once.
- 2. If you invest \$500 for one year at 7% interest, what is your balance after one year?
- 3. If you invest \$500 for two years at 7% interest, what is your balance after two years?
- 4. If your invest \$500 for three years at 7% interest, what is your balance after three years?
- 5. If you invest \$500 for 18 years at 7% interest, what is your balance after 18 years?
- 6. Assume you invest \$500 at 7% interest. Let b_n be the balance after *n* years.
 - What is the relationship between b_n and b_{n+1} ?
 - Is the sequence arithmetic or geometric?
 - Write a formula for b_n in terms of n.
 - How does your formula for b_n capture what you witnessed in Parts ??, ??, and ???
 - Write a formula in cell **G13** that calculates the ending balance. After you write the formula in **G13**, highlight the cell again and drag the black box down to **G20**.
 - Use this to answer the questions below.
- 7. Compare your balance after 18 years if you invested \$500 versus \$1000 at 7% interest. Your initial investment doubled. Did your ending balance double?
- 8. Compare your balance after 18 years if you invested \$500 at 7% interest versus at 14% interest. Your rate of return doubled. Did your ending balance double?
- 9. Compare your balance if you invested \$500 at 7% for 18 years versus the same investment for 9 years. The amount of time you invested has halved. Was your ending balance also reduced by half?

Reoccurring Deposits

Instead of just putting \$500 into an account and letting it grow, you may want to make reoccurring deposits.

- 1. If you initially invest \$500 and at the end of every year you deposit \$100 at 7% interest, what is your balance after one year? (check your answer with mine).
- 2. If you initially invest \$500 and at the end of every year you deposit \$100 at 7% interest, what is your balance after two years?
- 3. If you initially invest \$500 and at the end of every year you deposit \$100 at 7% interest, what is your balance after four years? (hint: you should calculate the balance after 3 years first)
- 4. If you initially invest \$500 and at the end of every year you deposit \$100, then the total amount of money deposited forms a sequence.
 - What type of sequence is this?
 - Write down the recursive relationship.
 - Write down an explicit formula for the total amount of money deposited after *n* years.
- 5. Assume you initially invest \$500 and at the end of every year you deposit \$100 at 7% interest. Let d_n be the balance after *n* years.
 - What is the relationship between d_n and d_{n+1} ?

- Use an Excel formula to calculate the balance after five years knowing the balance after three years.
- Drag this formula down to calculate the balance after 18 years.
- 6. Use the Excel technique developed in the previous question to compute the balance, if you initially invest \$500 and at the end of every year you deposit \$200 at 7% interest, what is your balance after four years?

Tuition The current tuition and fee schedule for a freshmen at the University of Utah taking 12 credits is shown in the following chart:

Utah Resident\$3,140.87Non-Utah Resident\$9,942.30

- 1. How many 12 credit semesters will you need to take to finish your major?
- 2. How much money will this cost (assume that tuition is constant)?
- 3. Use the Excel spreadsheet **Basic Savings Plan** you created to find how much money you would need to invest at 7% for it to be worth what you calculated tuition will cost after 18 years. A guess and check method works here; try to get within \$20.
- 4. Use the Excel spreadsheet **Reoccurring Deposits** to find an initial investment and a reoccurring deposit that will provide the balance you need after 20 years. A guess and check method works here; try to get within \$20.