Preface

Optimal Design, Structures, and Composites

This book discusses problems of structural optimization. The problem is to lay out several materials throughout a given domain to maximize or minimize an integral functional associated with the conductive or elastic state of an assembled medium. We assumed that several materials are available, and one is asked to arrange them on the volume of the body of a given shape. It turns out that the materials in the optimal body are mixed on an infinitely fine scale: The finer the scale, the better the construction. From an engineering point of view, optimization problems require the use of composites of given materials rather than materials singly.

As a rule, an optimal design is made of composites. Physically speaking, we use composites in designs because we prefer materials with properties that are not immediately available but can be obtained by mixing available materials; such a mixture can be more suitable than any of the individual ingredients. For example, composites assembled of isotropic materials can be anisotropic. Moreover, they can possess such exotic features as a negative thermal expansion coefficient, or a negative Poisson ratio. These and similar unusual features could be useful for solving optimization problems.

Optimal composites correspond to rapidly oscillating state variables, such as stresses and strains in elasticity or currents and fields in conductivity. The oscillation of optimal solutions is well understood in the theory of one-dimensional control problems. In some problems, the solution has to zigzag to satisfy the optimality requirement. The functional decreases as the zigzags become more finely scaled. It is not surprising that such generalized

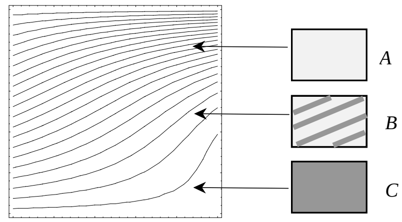

FIGURE P.1. The scheme of a composite structure that transforms the homogeneous boundary potential into an inhomogeneous boundary current. The horizontal sides are insulated, and the potential on each vertical side is constant. The current lines are shown. The inhomogeneity of the current is caused by the inhomogeneity of the material layout. The good conductor A attracts the current, the bad conductor C pushes the current away, and the anisotropic composite B turns the current in a desired direction.

controls also appear in the multidimensional problems of optimal layout of materials; here they correspond to microstructures of composites. Investigation of multidimensional optimization problems requires determination of the geometry of optimal composite structures. The one-dimensional analogue of the problem of the best microstructure is relatively simple because the only way to form a mixture in one dimension is to alternate materials along the line.

Example P.1 Let us consider the problem of an optimal inhomogeneous conducting structure that transforms the given boundary potentials to the desired boundary currents, as shown in Figure P.1P. Suppose that one has a set of materials of different isotropic conductivity and the layout of materials in the designed domain must be optimized. Clearly, one can control the boundary currents by varying the materials' layout, because the variation in conductivity forces the current out of regions of low conductivity and attracts it into regions of high conductivity. Careful consideration shows an additional mechanism of control through the use of anisotropic materials. The current is controlled and sent in the desired direction by refraction in an anisotropic composite. The last mechanism is specific to multidimensional problems and has no one-dimensional analogues. It shows the usefulness of the anisotropic composite media assembled of initially isotropic materials.

The use of anisotropy to control a process in a medium is well known. Observe a skier on a slope. The skier can control the direction of his motion because the resistance to sliding along the ski is much less than the resistance to sliding in the orthogonal direction. This mechanism allows the skier to traverse across the slope and make turns. Anisotropy is also used to steer a sailboat in a direction different from the direction of the wind. When a current of passive particles moves in a medium due to an applied

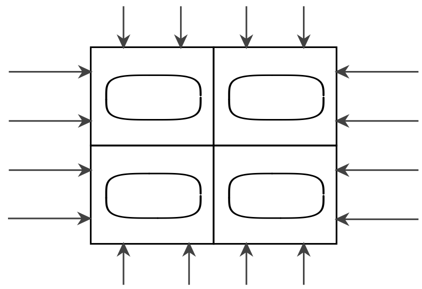

FIGURE P.2. The scheme of an elastic structure with cavities showing the maximum stiffness under a given loading. The intensity of the loading is anisotropic, and so is the corresponding optimal structure. Greater intensity of the loading corresponds to the direction of greater stiffness.

force field, the anisotropy of the medium plays a similar control role: It generates a current in a direction different from the direction of the force.

In the optimization of elastic designs, we also find intuitive reasons for using an anisotropic composite rather than isotropic materials.

Example P.2 Let us consider an elastic material that shows maximal stiffness under some anisotropic external loading; see Figure P.2P. One would assume that the larger the stress, the more stiffness is needed to resist. Therefore, we anticipate that the structure tends to be stiffer in the direction of a larger stress, even at the expense of being weakened in the direction of a smaller stress. Hence, we expect that anisotropic composites with controllable degrees of anisotropy are more suitable than isotropic materials for maximization of the stiffness.

Generally, structural optimization determines the structure, that is best adapted to the object of the design and the loading conditions. The adaptation implies uniform exploitation of the material. For example, the stiffness optimization is achieved by a structure that evenly stresses the material inside the structure. To keep the stress level constant, the fine-scale geometrical parameters of an optimal structure vary from one point to another. Sometimes, one needs to organize the layout in several length scales to optimize a structure, as seen in the structure of bones, leaves, airplane wings, or domes.

Structural Optimization in Engineering and Mathematics

In practice, the process of design always includes a mysterious element: The designer chooses the shape and materials for the construction using intuition and experience. Since ancient times this technique has proved effective, and for centuries engineering landmarks such as aqueducts, cathedrals, and ships were all built without mathematical or mechanical theories.

However, from the time of Galileo and Hooke, engineers and mathematicians have developed theories to determine stresses, deflections, currents and temperature inside structures. This information helps in the selection of a rational choice of structural elements. Certain principles of optimality are rooted in common sense. For example, one wants to equalize the stresses in a designed elastic construction by a proper choice of the layout of materials. The overstressed parts need more reinforcement, and the understressed parts can be lightened. These simple principles form a basis for rational construction of amazingly complicated mechanical structures, like bridges, skyscrapers, and cars. Still, knowledge of the stresses in a body is mostly used as a checking tool, parallel with the design proper, which remains the responsibility of the design engineer.

In the past few decades, it has become possible to turn the design process into algorithms thanks to advances in computer technology. Large contemporary projects require the use of computer-aided design systems. These systems often incorporate algorithms that gradually improve the initial design by a suitable variation of design variables, namely, the materials' cost and layout. Optimization techniques are used to effect changes in a design to make it stronger, lighter, or more reliable.

This progress has stimulated an interest in the mathematical foundations of structural optimization. These foundations are the main topic of this book. The theory of extremal problems is used to address problems of design. A design problem asks for the best geometry of layouts of different materials in a given domain. Of course, this approach simplifies (or, as a mathematician would say, idealizes) the real engineering problem, because questions such as convenience or cost of manufacturing are not considered. Analysis of optimal structures allows us to formulate general principles of an optimally designed construction. In particular, we can extend the intuitive principle of equally stressed construction to a multidimensional situation and find optimal structures that are, in a sense, hybrids of simple mechanisms.

The Purpose of the Book

A gap exists between mathematical approaches to variational problems and the practical use of results in structural optimization, theory of composites, and other engineering applications. On the one hand, we shall see how mathematicians develop advanced theories such as quasiconvexity and G-convergence for this purpose. On the other hand, the engineering and numerical community develops software for numerical optimization of complicated structures and successfully optimizes constructions of airplanes, bridges, and so on.

Progress in the area of numerical approaches is often ahead of mathematical methods required for an adequate formulation and rigorous solution to corresponding optimization problems. Mathematics deals with its own objectives: Standards of rigor are higher and models are simpler. This tends to make mathematical papers not too exciting for engineers. As usual, mathematicians use advanced methods to solve simple equations, and engineers use simple methods but work with complicated models. As a result, many practically oriented researchers are skeptical about the usefulness of refined mathematical theories. An opposite tendency, to interpret abstract mathematical results as prophecy, is no less risky.

These two approaches should be used in concert, each highlighting supplementary ideas of optimal design. I hope to present the foundations of structural optimization in a sufficiently simple form to make them available for practical use and to allow their critical appraisal for improving and adapting these results to specific models. I also hope that the reader will enjoy the beauty and elegance of the presented mathematical methods.

Often, mathematical analysis of an optimization problem leads to "unusual" solutions that are characterized by fractal geometries and are hardly suitable for manufacturing. This is acceptable in the framework of the chosen approach: We are looking for a mathematically correct solution, and we accept its features. From a practical point of view, the emergence of "strange" solutions reveals certain hidden features of optimality. These solutions should not be rejected as mathematical extravagance, but rather should be understood and interpreted in depth; often, they point to better solutions that may be approximated with available resources.

The Contents of the Book

Let us outline the contents of the five parts of the book.

Preliminaries. The exposition starts with an introductory Chapter 1 that discusses instabilities in one-dimensional variational problems. Specifically, we study variational problems with rapidly oscillating solutions and ways to describe these solutions. We also introduce the concept of relaxation of a nonstable variational problem by replacing the Lagrangian with its convex envelope.

Chapter 2 introduces the subject of optimization. We discuss conductivity of inhomogeneous materials and composites. The properties of a composite significantly depend on its microstructure. We introduce homogenization methods to describe the effective behavior of structures and calculate effective properties of special structures. Homogenization theory, in turn, puts forward the so-called G-closure problem (Chapter 3) that asks for bounds of effective properties of composites assembled from given materials. Bounds of G-closures correspond to composites of extreme effective properties that arise in optimal design.

Optimization of Conducting Composites. A large class of optimization problems of conducting composites requires only the simplest laminate structures for solution. These problems are used in the book as the testing ground for methods of structural optimization. We introduce all the control methods, including sufficient and necessary conditions of optimality and minimizing sequences. Chapter 4 deals with the optimization of the total conductivity of a domain. This problem does not have a classical solution; the optimal layout is a fine-scale mixture or a composite. We reformulate (relax) the problem, replacing the layout of available materials with the layout of optimal composites made of them. We also investigate the fields in optimal structures. Chapter 5 treats the problems of minimization of a large class of functionals associated with the solution to the conductivity problem, such as the minimization of the mean temperature in a part of the domain or the maximization of the boundary current.

Quasiconvexity and Relaxation. The second part deals with the relaxation technique of multidimensional variational problems with nonconvex integrands. This part contains most of the new mathematical results. In Chapter 6, we briefly discuss instabilities, the Weierstrass test, and we introduce the main tool for relaxation–the quasiconvex envelope.

In Chapter 7 we obtain upper bounds of the quasiconvex envelope by constructing some special minimizing sequences. The optimal layouts are represented by alternatng materials in laminate microstructures. We introduce special layouts with hierarchical geometries called "laminates of a high rank" and we derive their properties.

In Chapter 8 we derive lower bounds for the relaxed functional that correspond to sufficient conditions of optimality. The lower bound is built by a so-called translation method. We develop this method using the theories of quasiconvexity and compensated compactness.

In Chapter 9 we develop a technique of minimal extensions based on necessary conditions of the Weierstrass type. The extension we obtain gives an upper bound for the functional but avoids the explicit consideration of minimizing sequences.

All of these three approaches are illustrated by the solution of an optimization problem of a conducting structure that minimizes a sum of energies caused by several external sources.

G-Closures. To find the optimal structure of a composite, one first describes the set of effective properties of all possible microstructures. This set is called the G-closure of the properties of initially given materials. The fourth part discusses the knotty problems of G-closures. Chapter 10 deals with techniques used to describe the boundaries of the closures, i.e., the extreme effective properties of composites. The techniques are based on the variational methods introduced in Part III.

In Chapter 11 several examples of G-closures are constructed. These include the G-closures of conducting materials, the exact coupled bounds for conducting properties of composites, and bounds for properties of polycrystals.

Chapter 12 discusses multimaterial composites. The methods for these problems are less developed and more diverse. In particular, the technique of necessary conditions allows us to address the problem of bounds for a three-material composite.

Chapter 13 deals with the problem of complex conductivity. We suggest a variational principle for this problem, and we apply the variational technique to find coupled bounds on the real and imaginary parts of conductivity tensor.

Optimization of Elastic Structures. The last part of the book deals with optimal design of elastic structures. We begin with a discussion of the equations and variational principles for elasticity of inhomogeneous media and the algebra of fourth-rank tensors of elastic moduli (Chapter 14). In this chapter we also derive effective properties of elastic composites.

In Chapter 15 we consider the problem of minimization of the compliance of an elastic body, exploiting its similarity to the problems discussed in earlier chapters; we obtain elastic structures of extreme stiffness. We also discuss optimization of the shapes of cavities.

In Chapter 16 we survey the results regarding bounds for elastic moduli. Specifically, we consider an isotropic composite of two isotropic materials (plane problem), and we describe the bounds on its shear and bulk moduli. These bounds are coupled. We also consider the problem of isotropic polycrystals with extreme properties and describe the fractal geometry of optimal polycrystals. These examples demonstrate advanced applications of the variational technique described in Part III.

Chapter 17 discusses new formulations of a number of problems of structural optimization. We consider the minimization of the sum of elastic energies of different processes, the optimization of a periodic composite, the optimization of a nonenergetic functional, and the optimization in an unknown class of loadings. This last problem is formulated as a min-max game between the applied loadings and the responding structure.

Mathematical Methods

Mathematically, the book considers one type of problems in different settings. We describe optimal solutions to unstable variational problems. The goal is to define a solution that is reasonably smooth; particularly, it should not depend on the mesh in a discretization scheme. However, it often turns out that the optimal solution is characterized instead by infinitely fine oscillations. Special tests are developed to distinguish variational problems with smooth and nonsmooth solutions, and suitable frameworks for describing the solution with fine oscillations are worked out.

Both aspects deal with a special property of Lagrangians of the variational problem called quasiconvexity. Variational problems with quasiconvex Lagrangians possess stable solutions and problems with nonquasiconvex Lagrangians may not. Therefore, the test for oscillatory solutions requires consideration of the quasiconvexity of the Lagrangian. For one-dimensional variational problems and for some multidimensional problems, quasiconvexity degenerates to convexity, which makes the determination easy. Generally, however, the property of quasiconvexity is not geometric, and we need more refined tools to determine that a Lagrangian is quasiconvex.

If the Lagrangian lacks quasiconvexity, the minimizers generally are replaced with oscillating minimizing sequences. We perform the relaxation of the problem, also called the minimal extension, by averaging the solution over an infinitesimal volume. This corresponds to replacing the original nonquasiconvex Lagrangian with its quasiconvex envelope. In this way we obtain a new variational problem that possesses the same cost as the original one, but its solution is smooth and equal to the mean value of the fast oscillatory solution.

If quasiconvexity degenerates to convexity, the convex envelope can be built by systematic geometrical methods. There is no systematic universal method for constructing quasiconvex envelopes, so we instead build two extensions of the original Lagrangian, one above and one below the quasiconvex envelope (Chapters 7–9). Sometimes, these extensions coincide, in which case the quasiconvex envelope is determined.

The technique of bounds is addressed three times: first, in the context of one-dimensional variational problems (Chapter 1), then for the simplest multidimensional problems with a scalar potential (Chapter 3), and then in the general case (Chapters 6–9) of multidimensional problems with several state variables. This technique is used many times to solve various problems of G-closure (Part IV) and optimal elastic structures (Part V).

Related Topics

The theory of structural optimization lies at a busy intersection of several mathematical disciplines—optimal control, calculus of variations, homogenization, convex analysis—and is strongly influenced by materials science. Its applications include traditional optimal design, theory of composites, phase transition in solids, "smart" materials, nondestructive testing, self-organization in physics, biomaterials, and so on. Each of these fields has its own philosophy, its history, and a huge literature. Here we mention several of the related fields in mathematics and engineering. Each field could probably be identified by a representative, but not complete, list of the contributors. Specific references are placed in the body of the text.

The variational problems and problems of optimal control require methods of selecting and describing solutions with infinitely fast oscillations. It is known in control theory that minimization is generally achieved by an infinitely rapid oscillating control function, called the chattering control. This theory was originated by Pontryagin and Young and developed by Gamkrelidze, Krotov, Rozonoer, Varga, and others. The variational methods for nonconvex problems were introduced in works by Carathéodory, Morrey, and Young and developed in the works by Dacorogna, Ekeland, Kohn, Lions, Lurie, Müller, Murat, Raitum, Rockafellar, Strang, Tartar, Temam, and many others.

An average description of the layout for the highly oscillatory materials is the subject of the theory of homogenization. It was originated in the works by Babuška, Bakhvalov, Bensoussan, Hashin, Keller, Khruslov, Lions, Olejnik, Papanicolaou, Sanchez-Palencia, and Shtrikman, and developed in many respects in the works by Benveniste, Bergman, Bruno, Golden, Kohn, Kozlov, Markov, Milton, Norris, Panasenko, Telega, Torquato, Vigdergauz, Vogelius, Zhikov, and others. The advanced theories of solution to differential equations with rapidly oscillating coefficients can be found in the papers by Berlyand, Buttazzo, Cioranescu, Dal Maso, de Georgi, Fonseca, Francfort, Kinderlehrer, Kohn, Müller, Pedregal, Sukey, and Tartar, among others.

Approaches for bounds on the effective properties of composites are especially useful for our goals. This area, initiated around the beginning of the twentieth century by Rayleigh, Reuss, Voigt, and Wiener, was developed by Bruggeman, Hill, Hashin, Shtrikman, and Walpole and recently updated by Avellaneda, Benveniste, Beran, Francfort, Gibiansky, Kohn, Lurie, Markov, Milton, Murat, Nesi, Ponte Castañeda, Schulgasser, Talbot, Tartar, Torquato, Willis, and Zhikov, among others.

The physical side of the picture was highlighted by the mechanicians and applied mathematicians who formulated and solved structural optimization problems for several decades, starting from the works by Prager. We mention here the works of Armand, Arora, Banichuk, Bendsøe, Diaz, Eshenauer, Fuchs, Haber, Haftka, Kikuchi, Kirsch, Litvinov, Lipton, Mota Soares, Mroz, Olhoff, Pedersen, Rasmussen, Rozvany, Sigmund, Taylor, Tortorelli, and Zowe.

Computational techniques of structural optimization deserve special considerations, yet we feel that it does not fit the scope of this book, which is devoted exclusively to mathematical foundations of structural optimization. A detailed discussion of the computational techniques can be found, for example, in the books by Bendsøe, Haftka and Gürdal, Rozvany, and Papalambros and Wilde.

Natural Phenomena. Natural phase transitions, shape memory alloys, and naturally optimal biomaterials form a novel area of application of the discussed mathematical techniques. These problems, involving complicated materials, are in many respects similar to structural optimization. In both cases one deals with several materials or solid phases that are distributed in a domain in a specific way. The optimality requirement posed by a designer is parallel to a natural variational principle of minimization of the total energy of the system (the Gibbs principle). The transformation from one phase to another is parallel to the use of different materials in a design. In minimizing its energy, a natural system exhibits phase separation and forms a sort of natural composite that possesses optimal microstructure.

These similarities suggest that corresponding approaches could be applied to describe natural mixtures with minimal energy. This concept was put forward in the works of Ericksen, Khachaturyan, and Kinderlehrer and developed in the works of Ball, Bhattarcharya, Kohn, Fonseca, Grinfield, James, Luskin, Roitburd, Rosakis, Truskinovsky, and others. The methods of quasiconvexity are successively implemented for an explanation of structures arriving at some natural phase transitions; we refer to the works of the above-mentioned authors.

However, natural phenomena are much deeper than the problems of structural optimization. Indeed, the best engineering system should reach the global minimum of the minimizing functional that represents the quality of the system. On the contrary, an equilibrium state of a natural system corresponds to any local minimum of the energy. The energy of complicated natural systems is typically characterized by a large class of metastable local minima.

There are other differences, too. Contrary to an optimal engineering construction, a realizable equilibrium state of a natural system corresponds to a dynamical process that has led to it. Finally, natural composites usually are a random mixture of the states that correspond to local minima. The search for a distribution of local minima requires different techniques from those discussed here; we do not touch on this subject in the book.

Biomaterials. The amazing rationality of biological "constructions" also calls for the use of mathematical methods of structural optimization to model them. Consider, for example, the problem of the structure of a bone. A bone is a mechanical structure made of composites with variable parameters that adapts itself to its working conditions. It performs the clear mechanical task of supporting the organism. These features are similar to such man-made composite structures as masts, bridges, and towers. Therefore, it would be natural to apply optimization methods developed for engineering constructions to bone structures.

However, the two problems are not the same. In addition to the problems of local minima, stable evolutionary dynamics, and randomness already mentioned, it is not clear what quantity is minimized in natural evolutionary biomaterials (we mean the explicit optimality criterion of a natural structure, not a general reference to the evolution that perfects organisms). In engineering problems, the goal is the minimization of a given functional that is not the subject of a search or even a discussion. The problem is to find the structure that minimizes a functional prescribed by a designer. On the other hand, the structure of a bone is known, but it is not clear in what sense (if any) the bone structure is optimal.

The corresponding problem is the search for the cost functional of an optimization problem with a known solution. This problem has not been sufficiently investigated, to our knowledge.

Indexes, References, etc.

The electronic version of the manuscript for the book was prepared with the help of Professor Nelson Beebe using special BibTeX and LaTeX macros that he developed. In addition to the detailed table of contents, it contains the list of figures, references, the author/editor index, and the index of topics. Each item in the references points to the pages on which the source was referred to. The references section is ordered alphabetically by the name of primary author.

The author/editor index refers to the pages that contain the reference. Boldface author names indicate primary authors, while names in Roman text are nonprimary authors.

The book's Web site, `http://www.math.utah.edu/books/vmso`, contains an expanded bibliography in BibTeX form, an errata list, and other related resources. Please email your comments to `cherk@math.utah.edu`.

Use in the Classroom

The book is an extended and edited version of the author's lecture notes for courses delivered at the University of Utah. The contents of the book may be used for a year-long graduate course for students in applied mathematics, science, and engineering. We do our best to keep the exposition simple and do not hesitate to sacrifice rigor in favor of vividness, and generality in favor of vigorous illustrations. The references point to more rigorous formulations. The problems for discussion are in the end of chapters. Some of them are simple exercises; the others require more serious analysis and can be used for course projects.

A course in calculus of variations may be based on the classical material (Gelfand and Fomin, 1963; Ewing, 1969; Weinstock, 1974), supplemented by Chapter 1 (nonconvex one-dimensional problems), Chapter 4 (an example of a variational problem for a distributed system), and Chapters 6–9 (relaxation of nonconvex multivariable problems), with examples from Chapters 10–12 (G-closures).

A course in homogenization may use chapters from the "homogenization" books (Bensoussan, Lions, and Papanicolaou, 1978; Jikov, Kozlov, and Oleĭnik, 1994) and Chapters 2 and 3 (conductivity, homogenization, G-closure), Chapter 7 (laminates, various structures of laminates of high rank), Chapter 12 (multiphase structures), and Chapter 14 (elasticity, homogenization and matrix laminates). Chapters 4 and 5 (optimization by laminates) may be used as examples of the use of composites.

A course in structural optimization may use Chapters 4 and 5 (optimization of conducting bodies), Chapters 6–9 (relaxation of nonconvex multivariable problems), and Chapters 14–17 (elasticity, optimization of elastic structures).

Credits

The author has been tempted to present his view of the history of structural optimization and nonconvex variational methods, but he has given that up because the topic is too awesome, controversial, and sensitive. The very length of the preceding list of names and topics testifies to this. We give credit here to authors in a specific context of themes discussed. It is hardly possible to give a complete survey of even the recent development of related topics: New branches of the theory are constantly appearing. Instead, we concentrate our attention on underlying ideas and methods that should help to find solutions to new problems.

Most of the results and opinions presented are based on or related to the author's research, conducted for the most part in collaboration with Tim Burns, Elena Cherkaeva, Andrey Fedorov, Leonid Gibiansky, Lars Krog, Konstantin Lurie, Graeme Milton, Robert Palais, and Shmul Vigdergauz.

Many new results obtained by Grégoire Allaire, Marco Avellaneda, Martin Bendsøe, John Ball, Gilles Francfort, Leonid Gibiansky, Zvi Hashin, Robert Kohn, Robert Lipton, Konstantin Lurie, Graeme Milton, François Murat, Vincenzo Nesi, Niels Olhoff, Ole Sigmund, Gil Strang, Vladimír Šverák, Luc Tartar, Salvatore Torquato, Smul Vigdergauz, Vasily Zhikov, and others are explicitly used in the text.

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Salt Lake City, Utah — Andrej Cherkaev
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