Case Studies in Mathematical Modeling—Ecology, Physiology, and Cell Biology

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PREFACE

As its name indicates, the field of mathematical biology is inherently interdisciplinary. Students and researchers seeking to enter this field, or to broaden their knowledge, face special challenges. How does one strike an appropriate balance between learning the details of the underlying biology and the intricacies of the mathematics? Are explorers in this area beset, like Odysseus, by the twin dangers of Scylla and Charybdis? Or are they at risk of being seduced by the sirens of biological complexity and mathematical elegance?

Books that seek to introduce this field must guide readers along a safe path between the same dangers. This volume finds the path not by following some pedagogical theory or by believing rumors about past shipwrecks, but by tracing the wake of successful researchers who have survived to tell the tale. Each scientist who has addressed biological questions with mathematical methods has found a different way, and this book presents this diversity. The case studies presented herein invite readers to join a researcher as he or she attacks a significant biological problem from start to finish. Each chapter combines the focus on cutting-edge research characteristic of the professional literature with the emphasis on teaching characteristic of a textbook. The authors provide a synthetic view of the biological problem, illustrating the multiple approaches attempted, and the strengths and weaknesses of each. The goal is to motivate and explain biological problems and their mathematical solution, while simultaneously exemplifying the process of developing a successful research program.

These case studies guide advanced undergraduates, beginning graduate students, and researchers along several such paths. Students who have worked to build mathematical skills will able to set sail in quest of important problems. The goal is to initiate them into both the diversity of approaches to mathematical biology and the breadth of the field. This book thus has two unique features, summarized as **case studies** in **mathematical biology**.

As a guide to both student and teacher, we suggest that the chapters can be read with the following general framework for modeling in mind.

1. The first step is to identify a biologically interesting problem which has a significant aspect that requires mathematical modeling, to identify critical observations on which to base a model, and to distinguish between dependable and undependable experimental results.

2. The second step is to formulate the model conceptually, making reasoned judgments regarding which processes to include and which to exclude.

3. The next step is to convert the conceptual model into a mathematical model and estimate parameters in the model, keeping in mind differing levels of certainty regarding their values.

4. The final step is to analyze the model, both qualitatively and perhaps numerically, if analytical solution is impossible, and use the results to interpret the original critical observations, make new predictions and propose new experiments.

This complex process cannot be distilled into an algorithm, because it is as much art as science. As in the experimental sciences, the techniques of modeling are learned by example and by hands-on experience. This book aims to provide both.

Coverage

Powerful new techniques in molecular biology, physiology, genetics and ecology are making biology more quantitative and more unified. Mathematical methods are needed both to analyze increasing volumes of data, and to forge connections between data shedding light on common problems from different angles.

Due to the expanding role of mathematics as a unifying force in biology, broad coverage of the field of biology is essential. The topics chosen in this volume fall into three overlapping categories: ecology, cell biology, and physiology. Although the areas have their characteristic concerns, each area uses mathematics to link levels of biological organization, depends on related mathematical techniques from dynamical systems, and, more broadly, emphasizes that mechanisms which act at the cellular or molecular level manifest themselves in the functioning of the whole organism in its environment. Each of the three sections of the book begins with an introduction by the editors that elucidates specific themes and concerns characteristic of that section.

How to use this book

We envision three primary uses for this book: as a supplementary text to accompany a mathematical biology course, as a primary text for an intensive mathematical modeling course, and as a reference volume for researchers and students.

The mathematical level of the book is graded, becoming more advanced in the later chapters. Every chapter requires that students be familiar and comfortable with differential equations and linear algebra (short appendices outlines relevant aspects of these techniques). Partial differential equations and functional differential equations are used more and more in the later chapters, and previous background or concurrent study is necessary for full comprehension. Numerous other techniques, ranging from stochastic processes to statistics, are used by the authors as needed. Like researchers, students must realize that research is not an idealized romance, with pre-defined problems and techniques that are “made for each other.” Rather, techniques must be developed and modified constantly, and we hope that students learn best when forced to learn the way researchers do.

These case studies are written with more concision and demands than an ordinary textbook. The chapters are thus ideally suited to serve as starting points for group or individual projects, providing sufficient background and explanation to stand on their own, but constantly motivating students to read alternative literature and experiment with novel mathematical methods. In conjunction with an ordinary textbook, students will see how standard topics are the foundation for working mathematical biologists, but that building on that foundation requires constant imagination and ingenuity to choose and modify the appropriate method.

As a stand-alone text, the book will be most appropriate for a more advanced course, where students have a strong background in ordinary and partial differential equations that they are yearning to put to use. Such students should find the book to be an amiable but insistent companion, a source of new ideas, and, at times, a source of creative irritation.

Acknowledgments

During the academic year 1995–96, the Department of Mathematics, in cooperation with the Departments of Biology, Bioengineering, Human Genetics and the Nora Eccles Harrison Cardiovascular Research and Training Institute at the University of Utah, ran a special educational program entitled "A Special Year in Mathematical Biology". This volume is the outcome of the lectures given during the special year. We are grateful to the lecturers, who contributed their time and energy to what proved to be a very worthwhile educational experiment, and then further agreed to record their lectures in the chapters herein.

We also gratefully acknowledge the financial assistance provided by the National Science Foundation, the Department of Mathematics, the College of Science, the Office of the Vice President for Research, the Departments of Biology, Bioengineering, Human Genetics and the Nora Eccles Harrison Cardiovascular Research and Training Institute at the University of Utah for making the Special Year, and thus this book, possible.

A large measure of the success of the program is due to the participation of visiting graduate students and the local graduate students and post-doctoral fellows. In addition to attending the course and seminars, the local graduate students and post-doctoral fellows created a pleasant ambiance for the visiting students and faculty, and acted as informal reviewers of the chapters. We thank Pat Corneli, Barry Eagan, Daniel Grunbaum Andrew Kuharsky, "Colonel" Tim Lewis, Eric Marland, Steve Parrish, Bradford Peercy, Steve Proulx, Pejman Rohani, Peter Spiro, Min Xie, Toshio Yoshikawa, and Haoyu Yu for their participation and work.

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